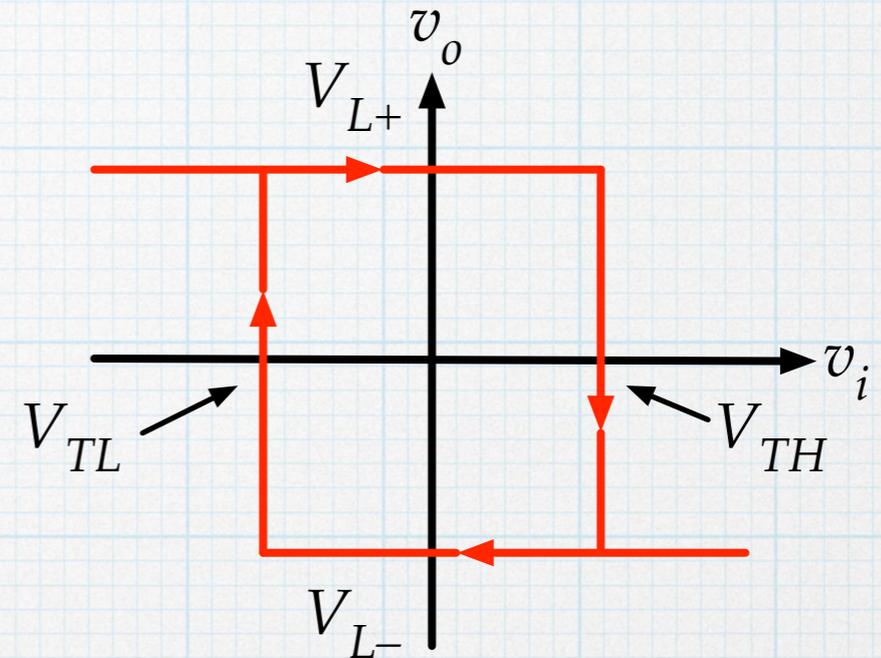
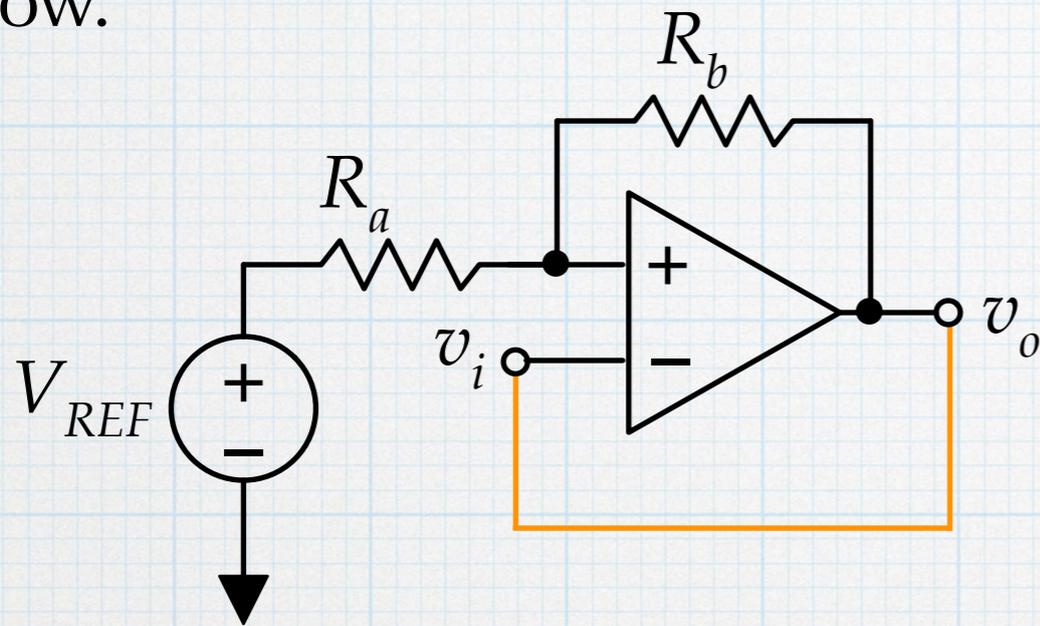


# non-linear oscillators

The inverting comparator operation can be summarized as “When the input is low, the output is high. When the input is high, the output is low.”

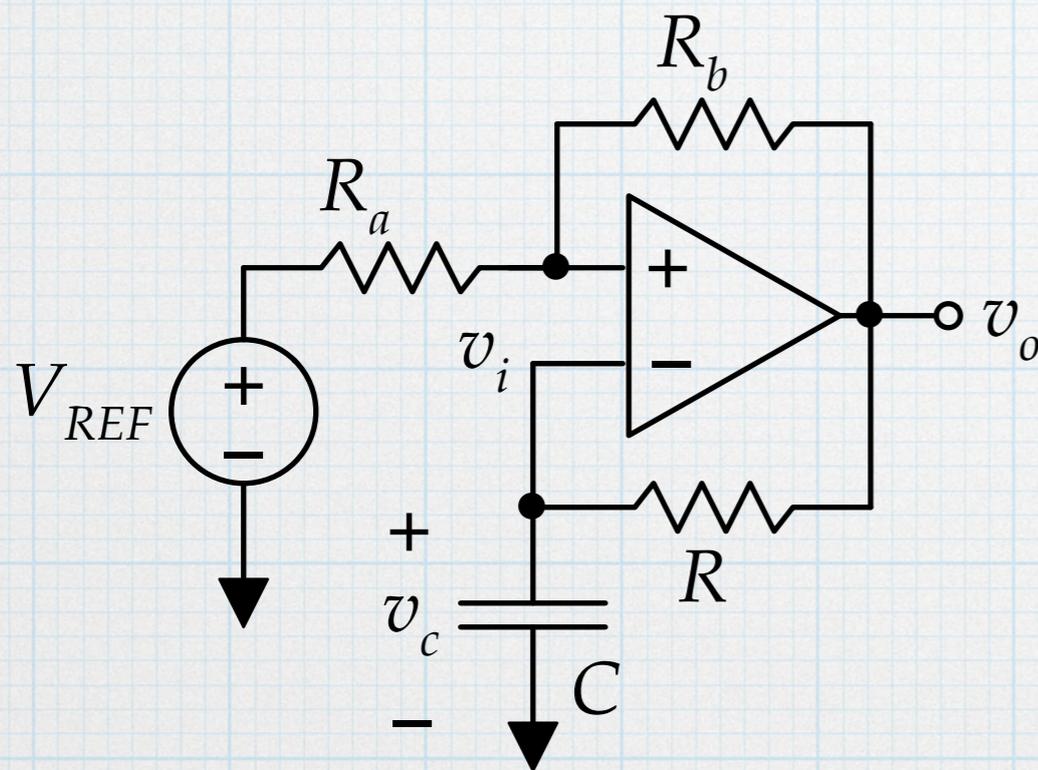


$V_{TL}$  and  $V_{TH}$  are given by the expressions derived in the comparator notes.

Fiendish question: What would happen if we tied the output back to the input?

The circuit would become “confused”. When the output was high, then input would be high. But with the input high, the output would go low, causing the input to go low. The comparator is faced with a conundrum, and it would respond by oscillating back forth between high and low as fast as it possibly could.

This business of switching back and forth might be useful. This would be another form of oscillator circuit. But to be practical, we need to put a time delay between when the output switches and when the input senses that the output has switched. We can do this by using an  $RC$  circuit to tie the output back to the input. The charging and discharging of the capacitor will determine the time delays and hence the oscillation frequency.



1. Suppose that the input is low and the output is high. The capacitor voltage is also low.

2. The output voltage will charge the capacitor.

3. As the capacitor charges, the input climbs higher.

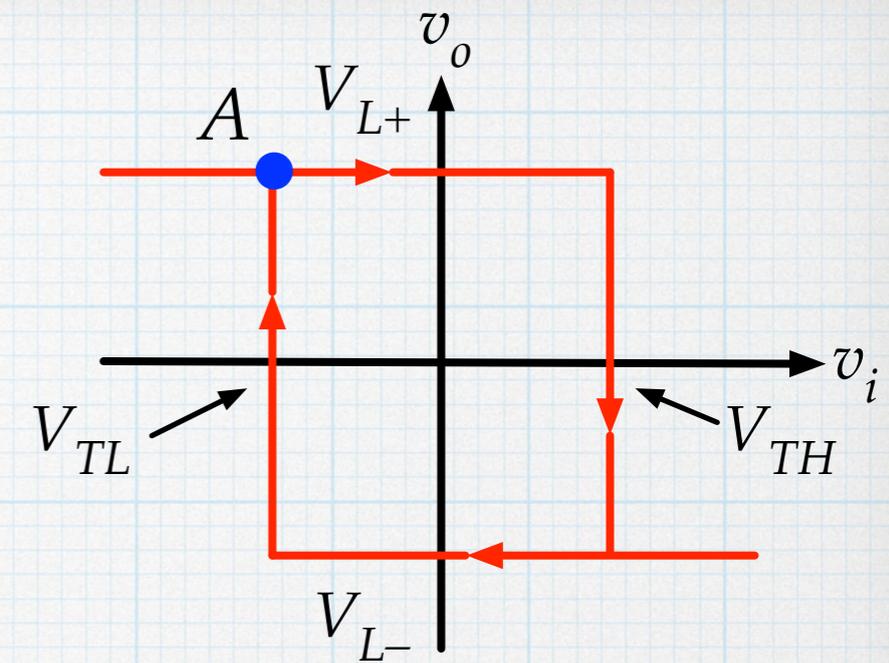
4. Once the input voltage becomes equal to  $V_{TH}$ , the comparator will flip – the output voltage goes low.

5. With the input low, the capacitor discharges, and  $v_i$  decreases.

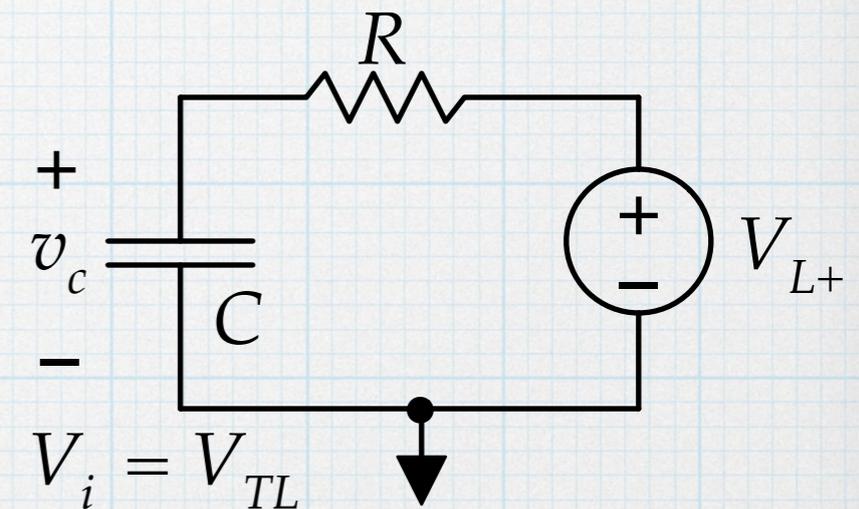
6. The input will decrease until it hits  $V_{TL}$ , causing the output to go high again. The cycle repeats.

# Output high

The comparator will always be operating at some point on its transfer characteristic. Let's start by assuming the output has just switched high, putting us at point  $A$  on the transfer characteristic. The output voltage is at  $V_{L+}$  and the input (capacitor) voltage is at  $V_{TL}$ .



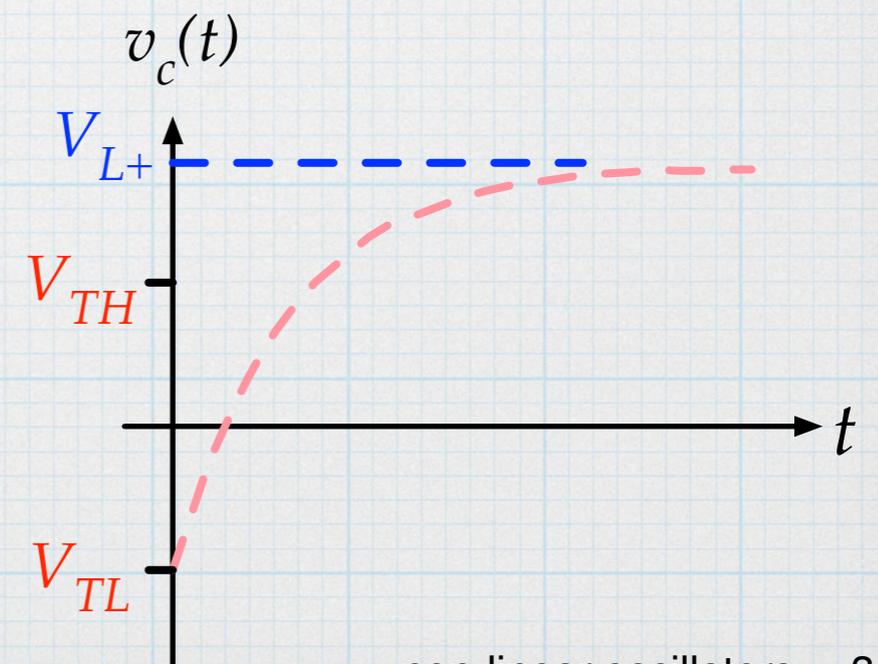
The capacitor and resistor form a simple  $RC$  circuit with a source voltage of  $V_{L+}$  and initial capacitor voltage of  $V_{TL}$ . The capacitor will begin charging.



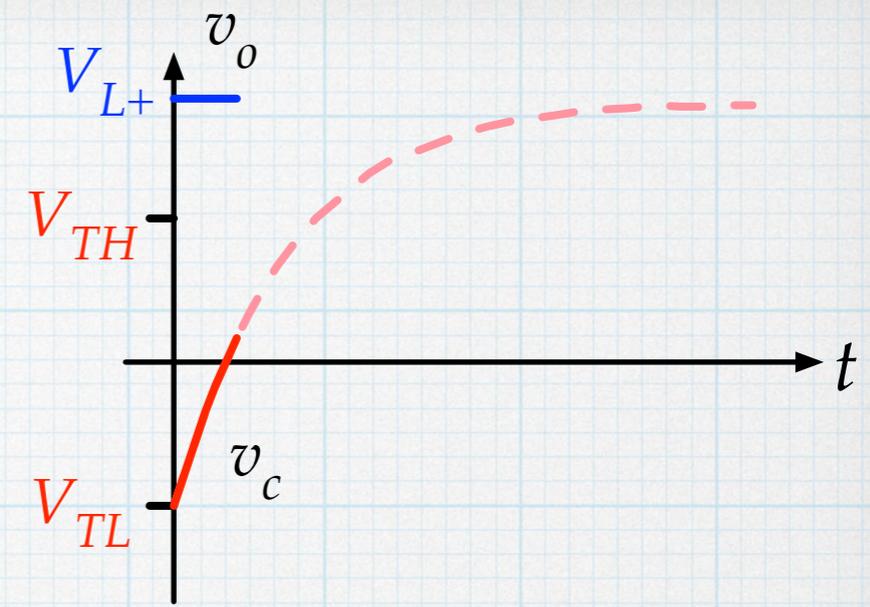
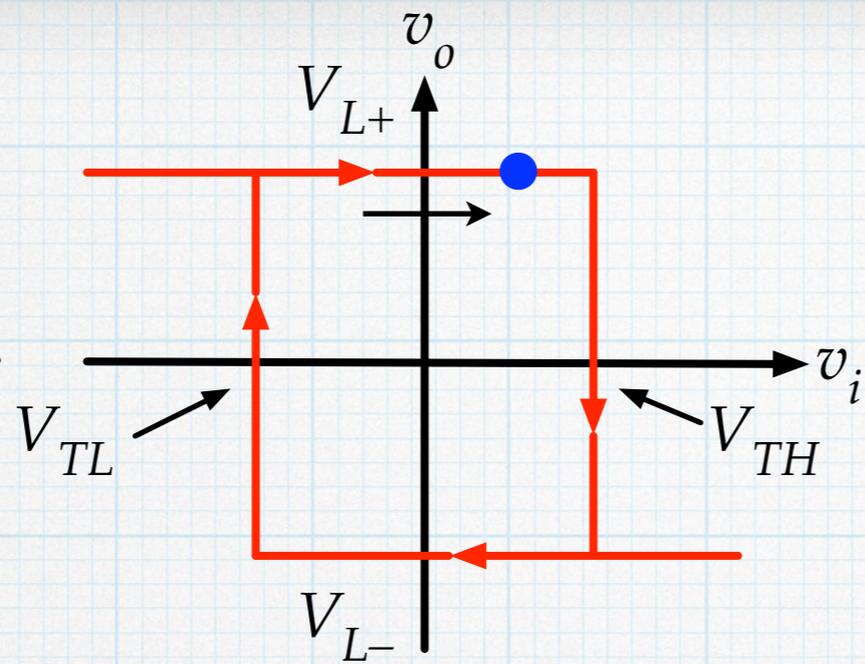
From EE 201:

$$v_c(t) = V_f - (V_f - V_i) \exp\left(-\frac{t}{RC}\right)$$

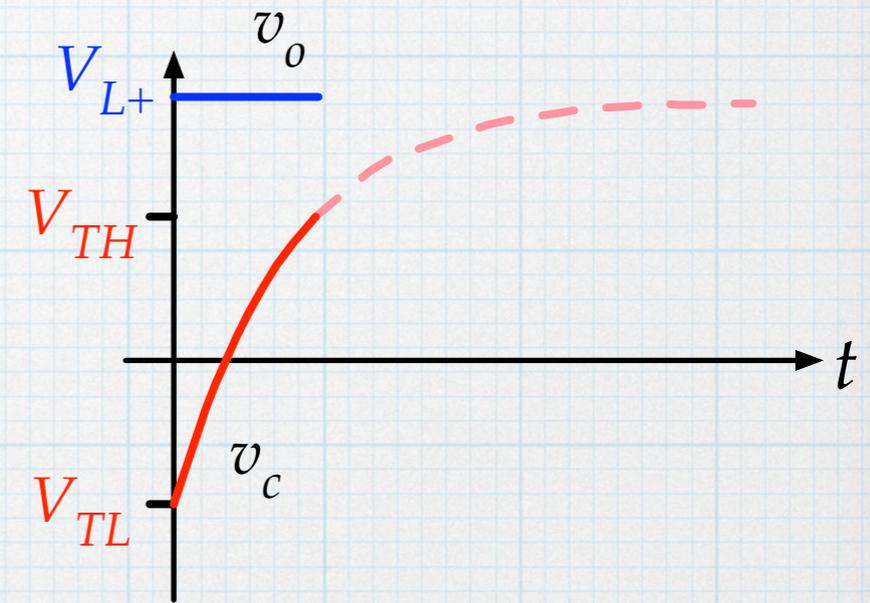
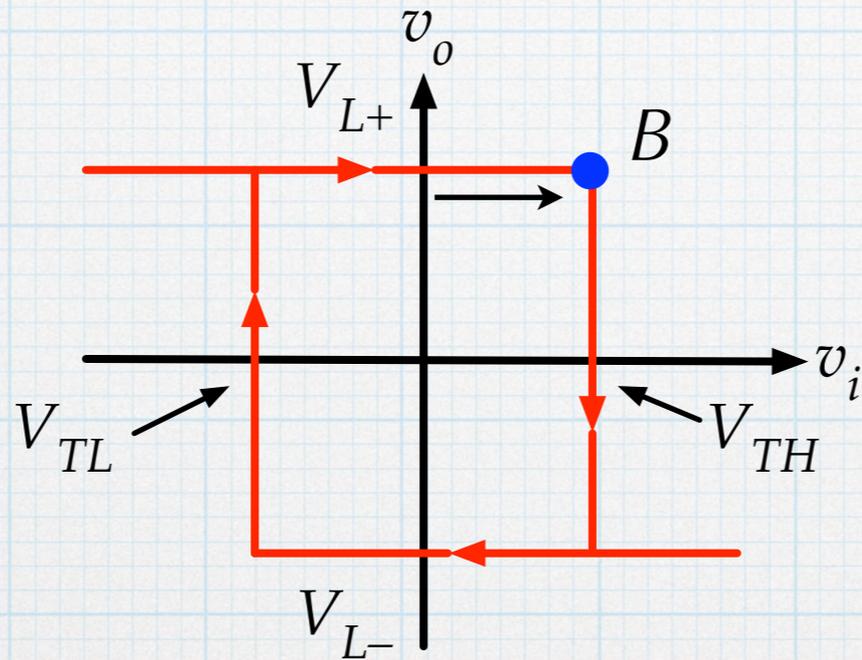
$$v_c(t) = V_{L+} - (V_{L+} - V_{TL}) \exp\left(-\frac{t}{RC}\right)$$



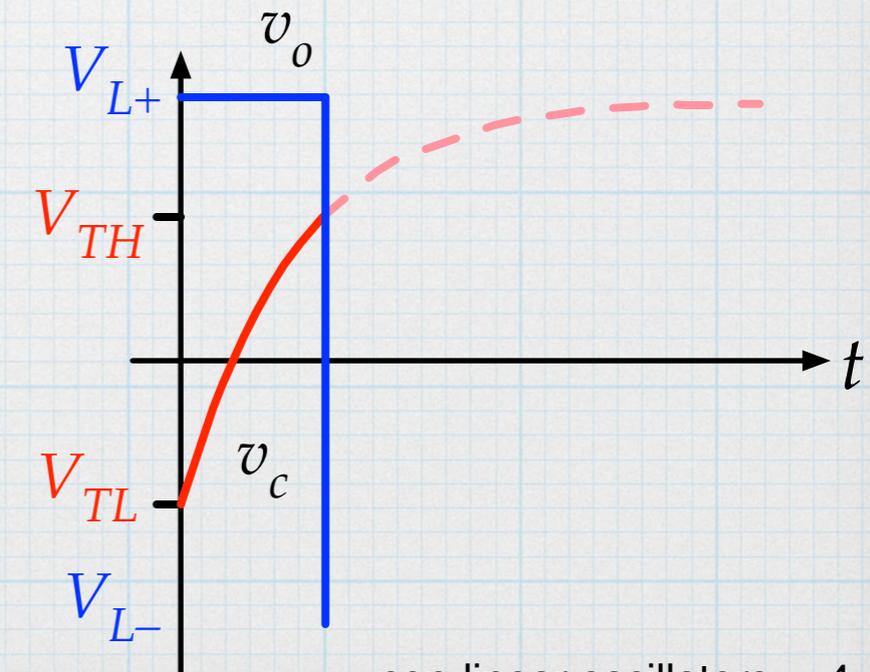
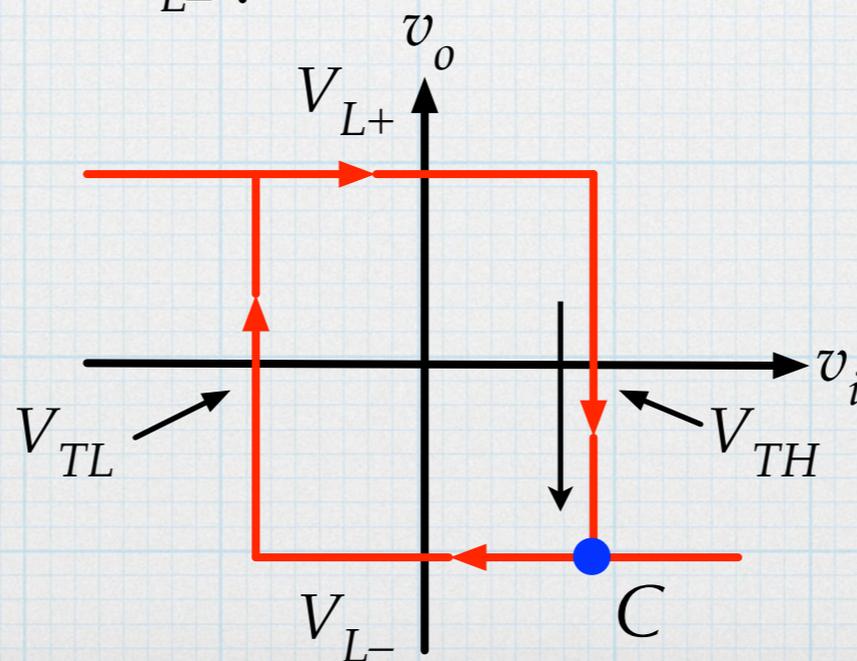
As the capacitor charges, the voltage increases...



...until  $v_c = V_{TH}$ ...



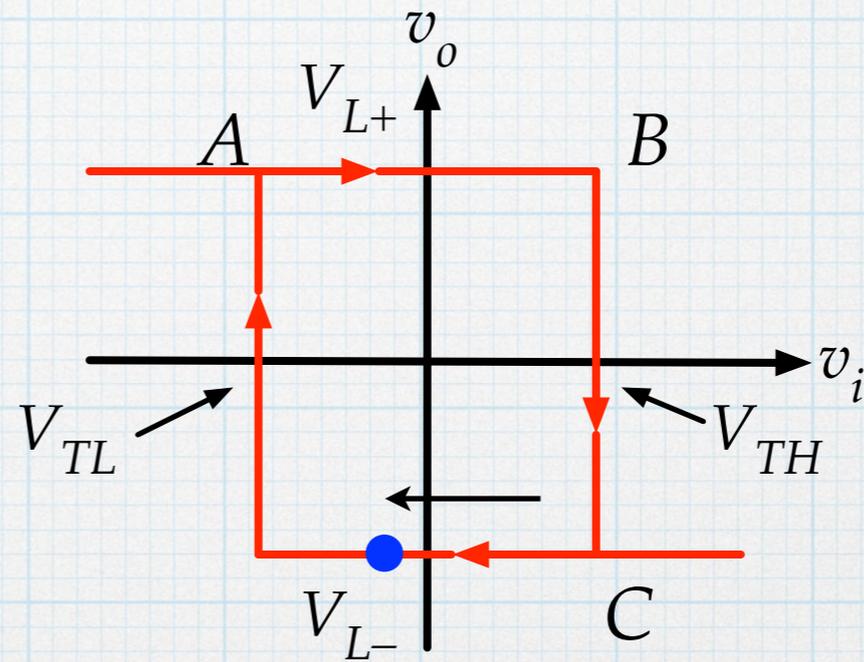
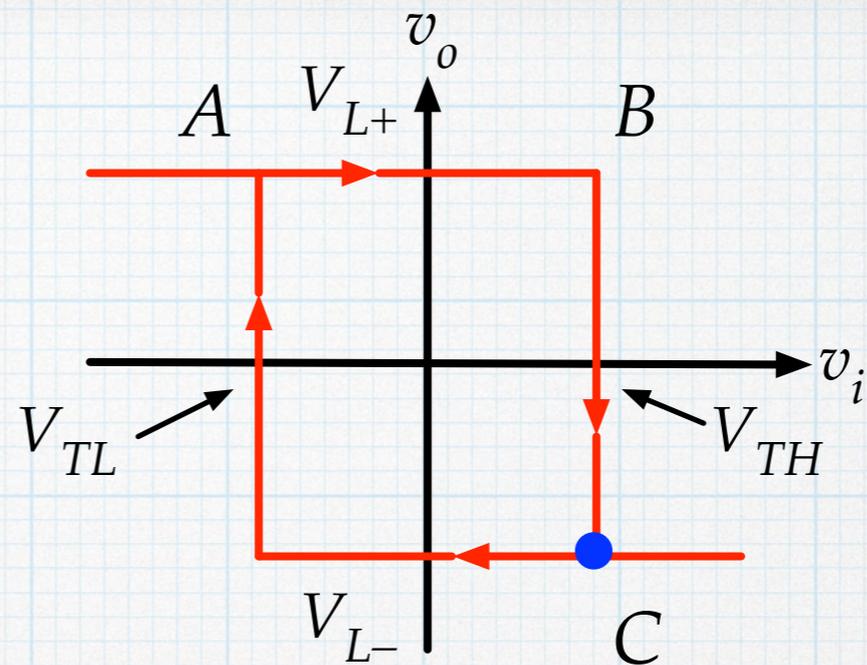
...which causes the comparator to switch to the low-output condition.



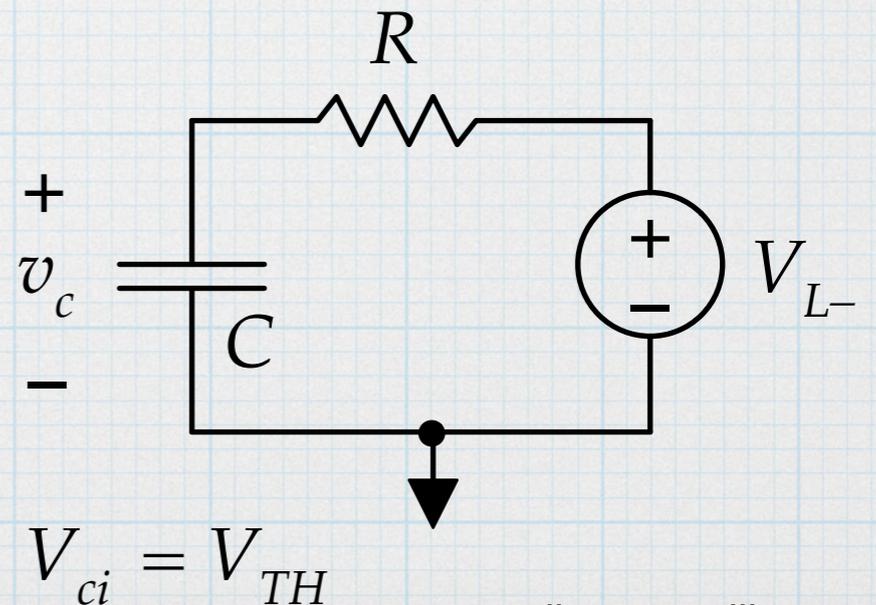
# Output low

Now the comparator is at point C on the transfer characteristic. The output voltage is low ( $V_{L-}$ ) and the capacitor voltage is high ( $V_{TH}$ ).

The difference in voltages causes the capacitor to *discharge*, moving the operating point along the lower branch of the curve.

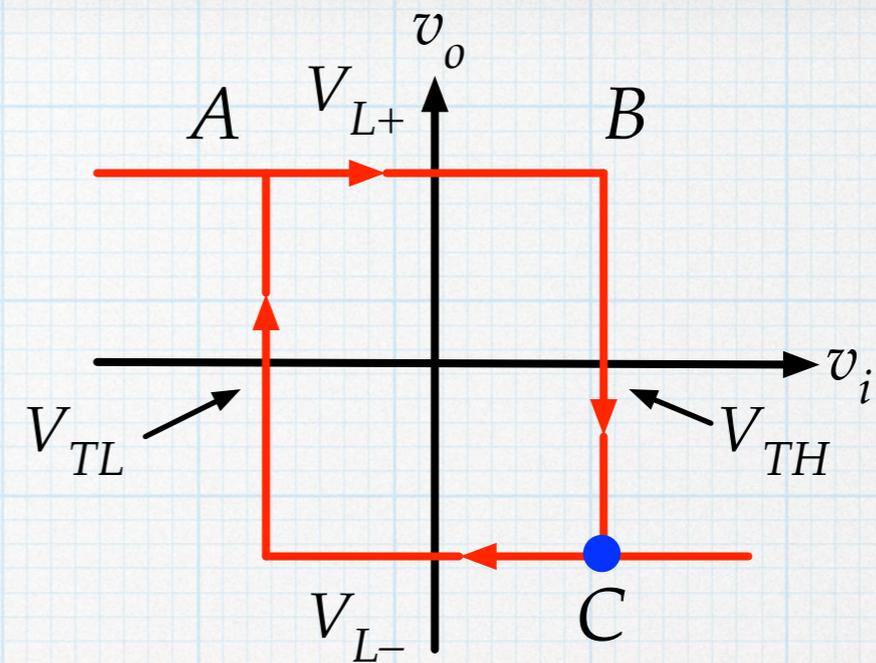


$$v_c(t) = V_{L-} - (V_{L-} - V_{TH}) \exp\left(-\frac{t}{RC}\right)$$



# Output low

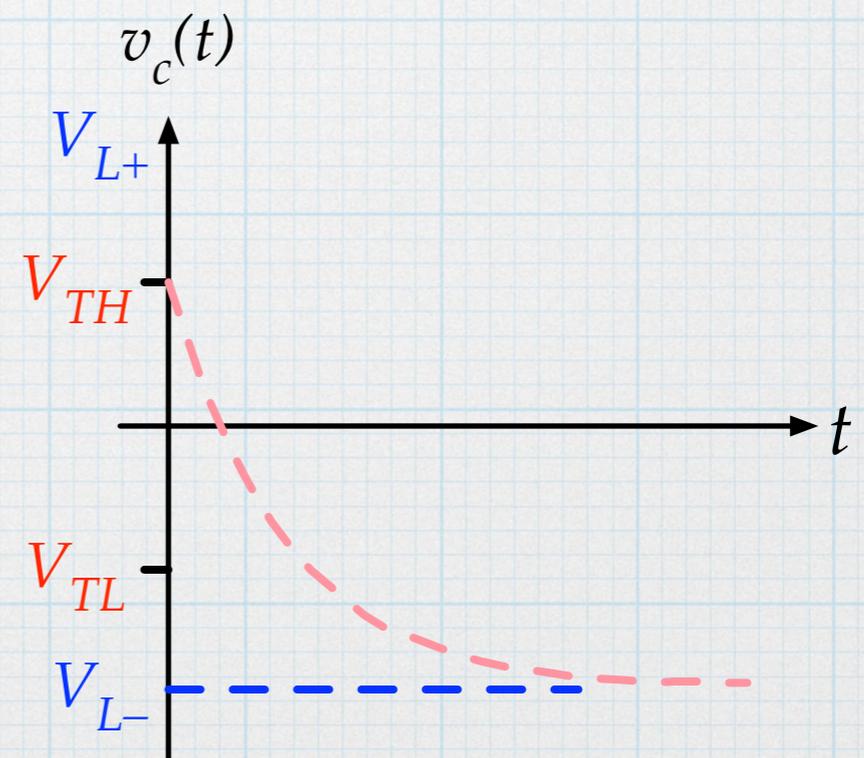
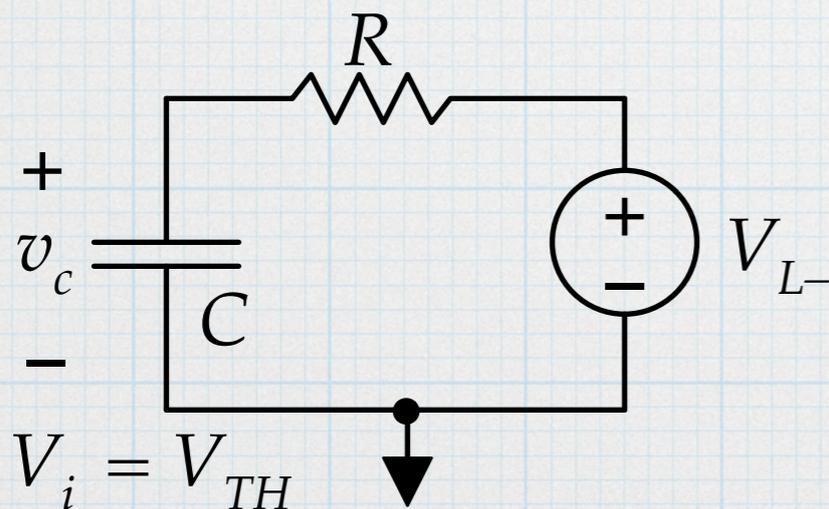
Now the comparator is at point C on the transfer characteristic. The output voltage is low ( $V_{L-}$ ) and the capacitor voltage is high ( $V_{TH}$ ).



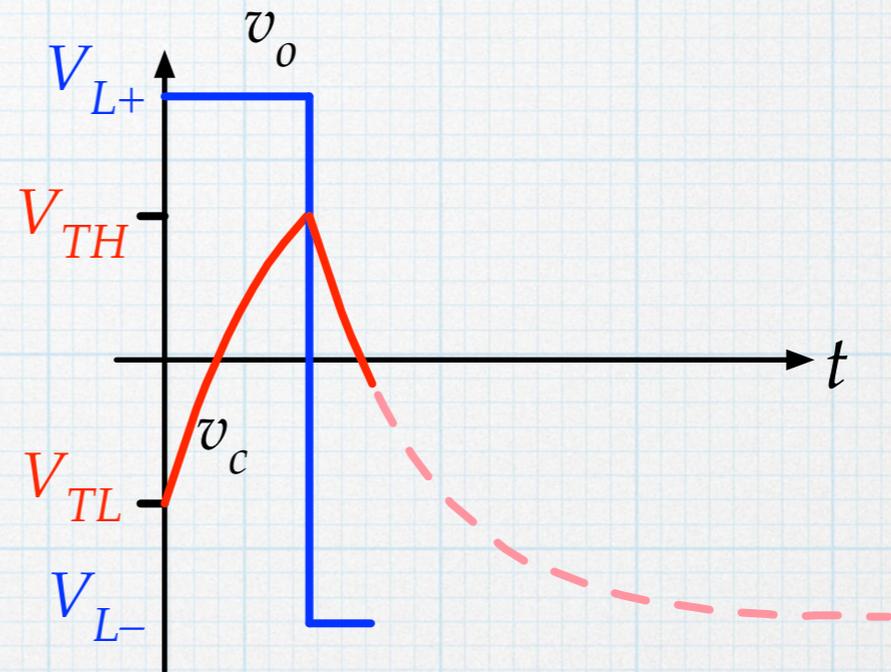
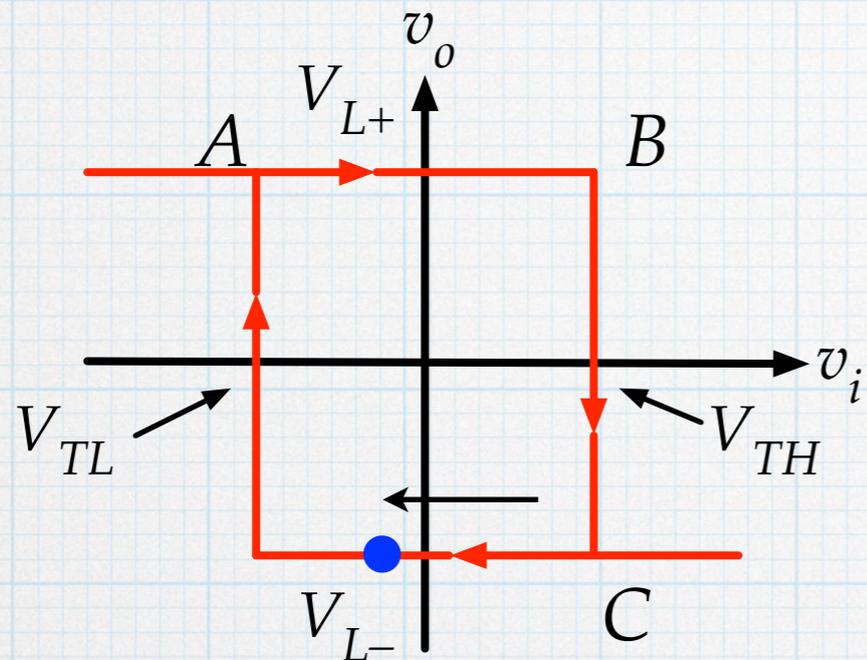
The difference in voltages will cause the capacitor to *discharge*.

$$v_c(t) = V_{L-} - (V_{L-} - V_{TH}) \exp\left(-\frac{t}{RC}\right)$$

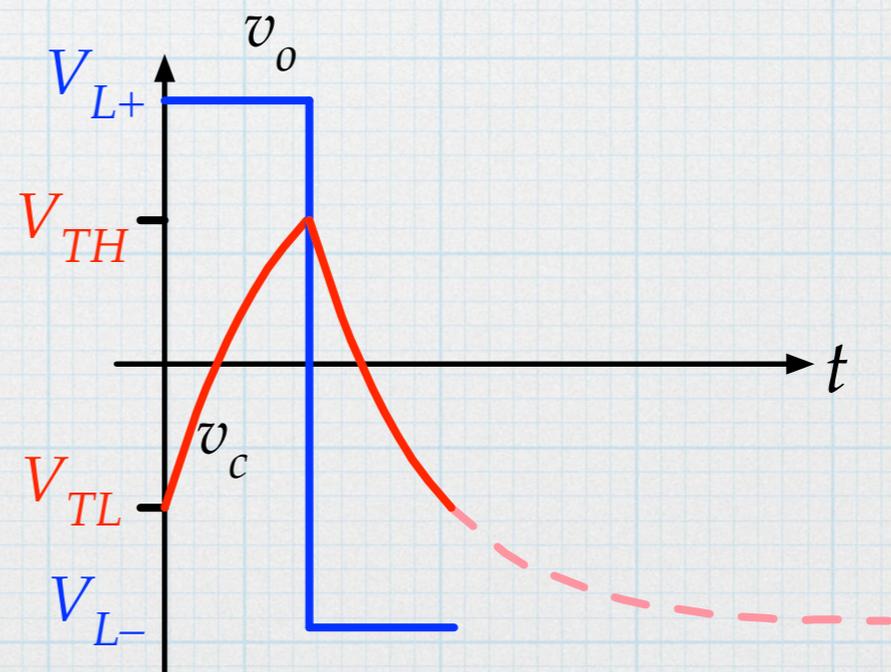
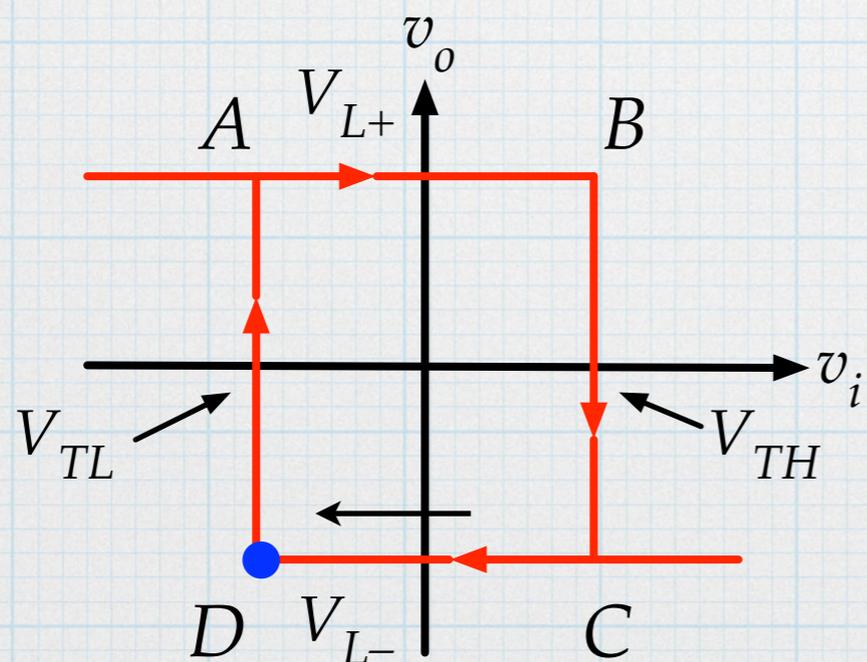
$$V_{L-} < V_{TH}.$$



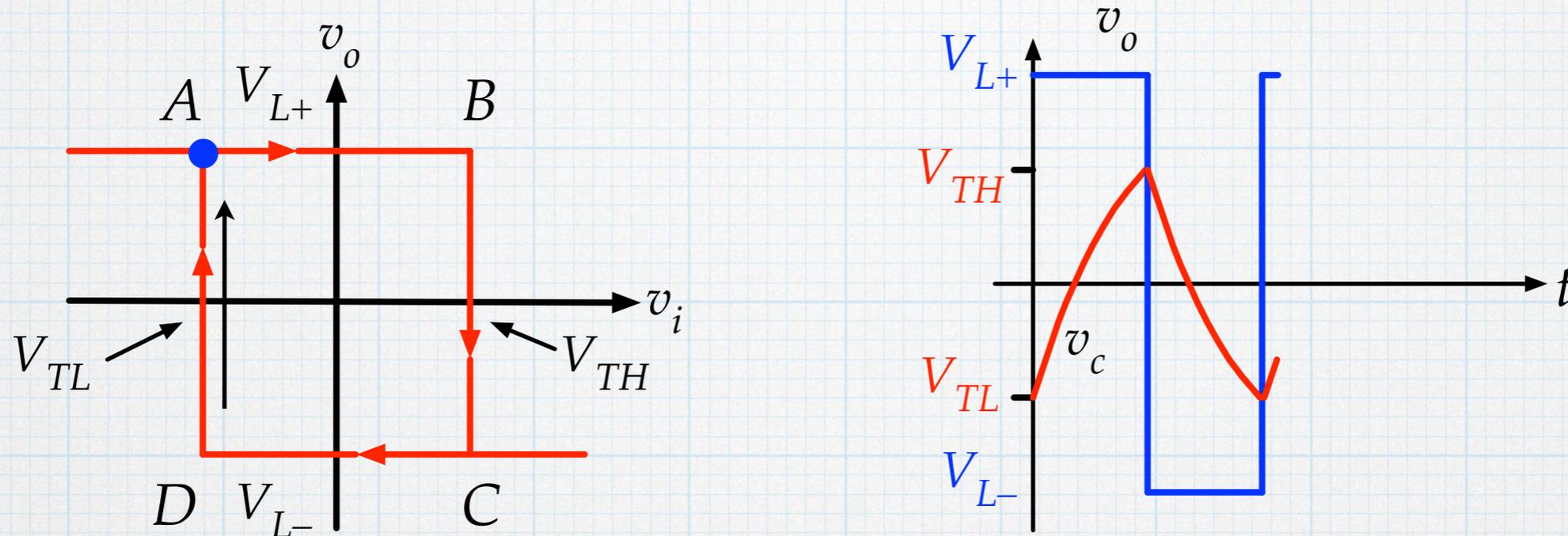
As the capacitor discharges, the comparator operating point moves along the lower branch of the transfer characteristic...



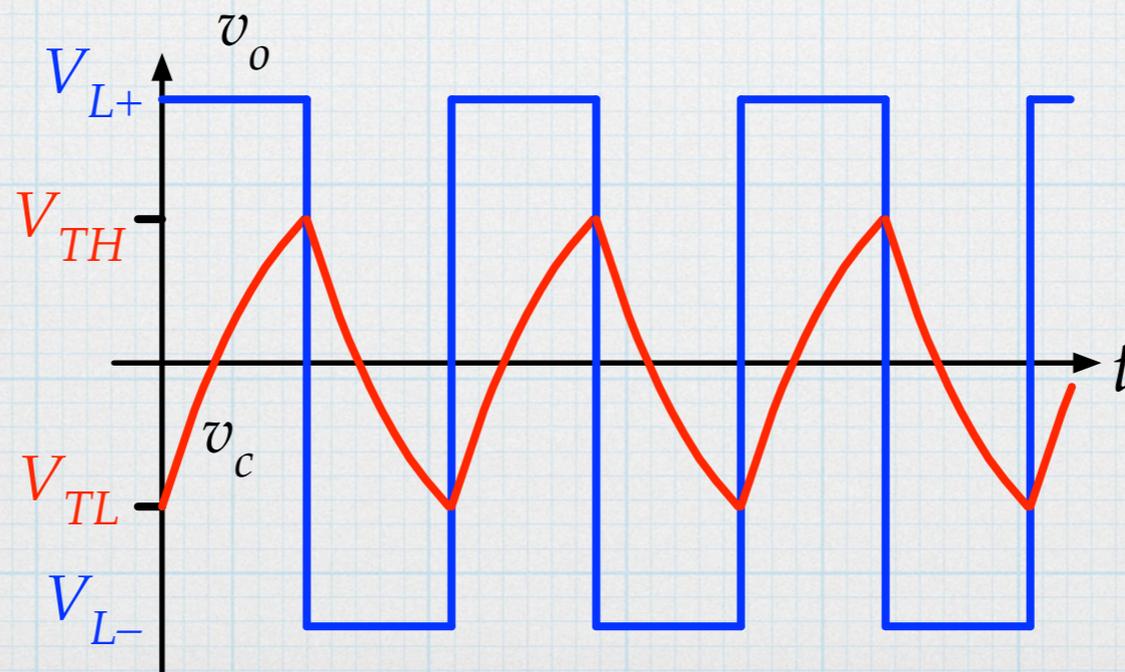
After some time, the capacitor voltage drops to  $V_{TL}$ ...



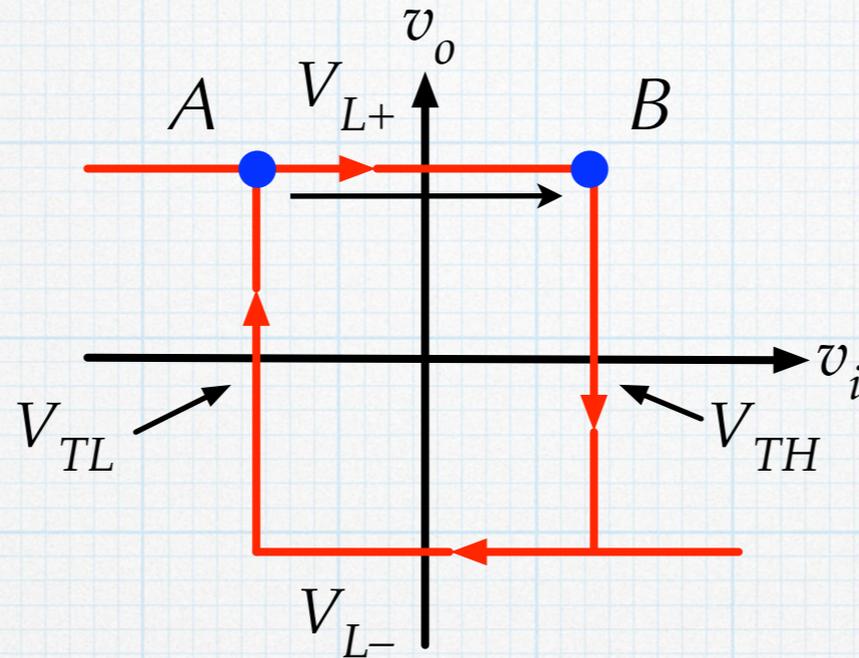
...which causes the comparator to switch abruptly to the high-output condition – right back to where we started.



The cycle starts over and repeats indefinitely. The capacitor charges and discharges between  $V_{TH}$  and  $V_{TL}$  and the output switches between  $V_{L+}$  and  $V_{L-}$ .



# Time high



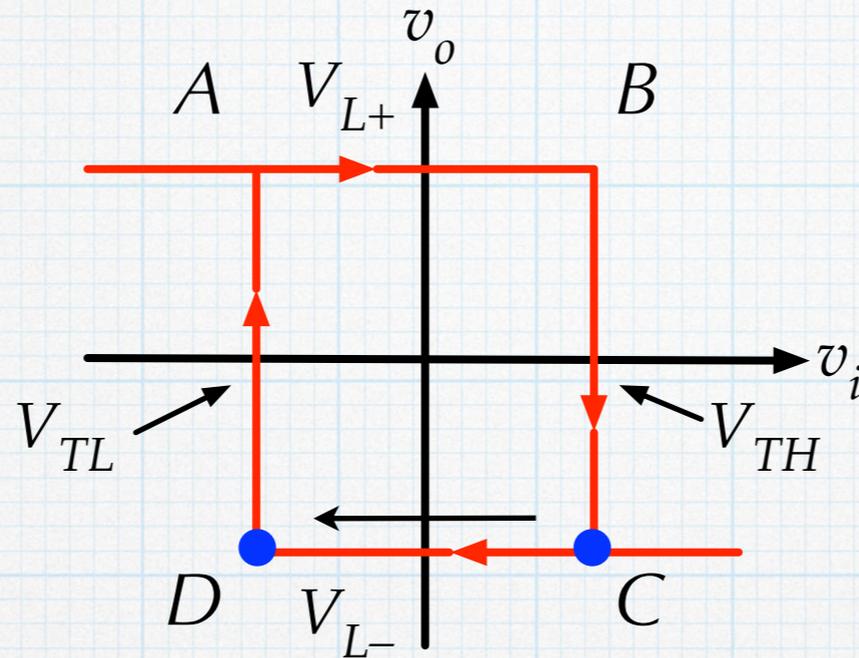
The time required to move from  $A$  to  $B$  on the characteristic is found using the capacitor charging equation. This is the time that the output will be high.

$$v_c(t) = V_{L+} - (V_{L+} - V_{TL}) \exp\left(-\frac{t}{RC}\right)$$

$$V_{TH} = V_{L+} - (V_{L+} - V_{TL}) \exp\left(-\frac{T_H}{RC}\right)$$

$$T_H = RC \ln\left(\frac{V_{L+} - V_{TL}}{V_{L+} - V_{TH}}\right)$$

# Time low

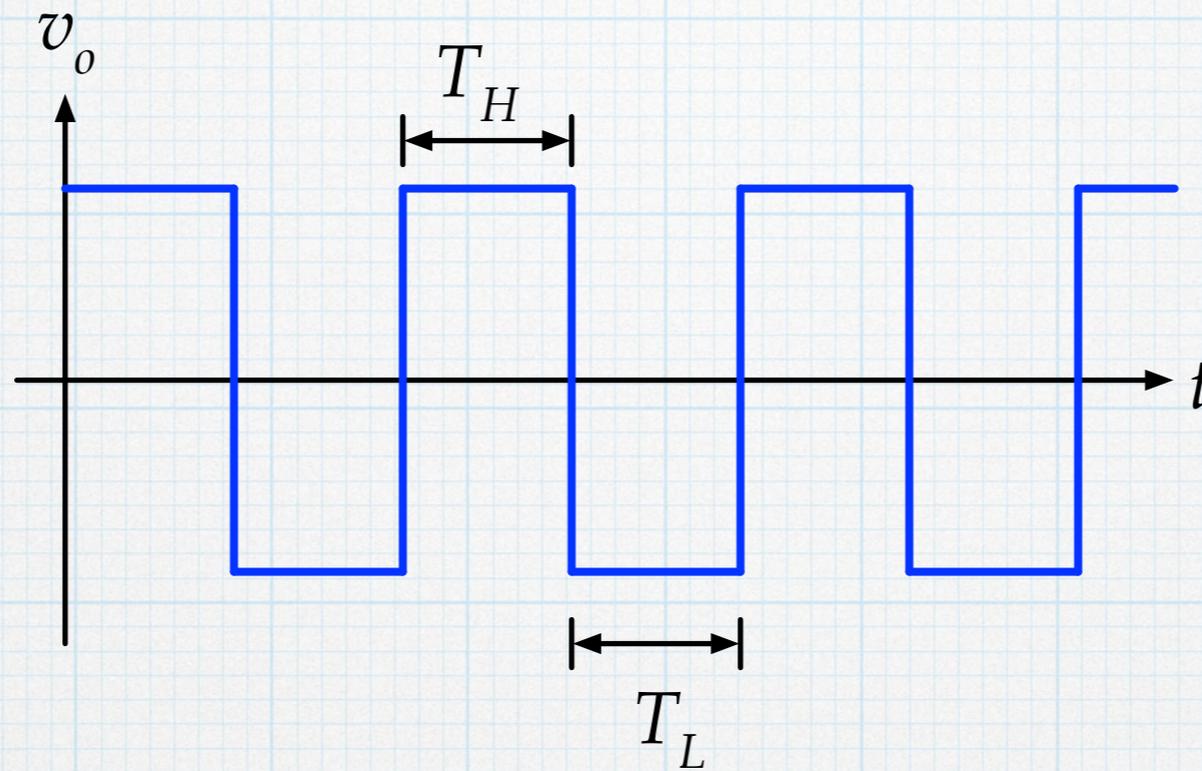


Again, the time required to move from  $C$  to  $D$  on the characteristic is found using the capacitor (dis)charging equation. This is the time that the output will be low.

$$v_c(t) = V_{S-} - (V_{S-} - V_{TH}) \exp\left(-\frac{t}{RC}\right)$$

$$V_{TL} = V_{S-} - (V_{S-} - V_{TH}) \exp\left(-\frac{T_L}{RC}\right)$$

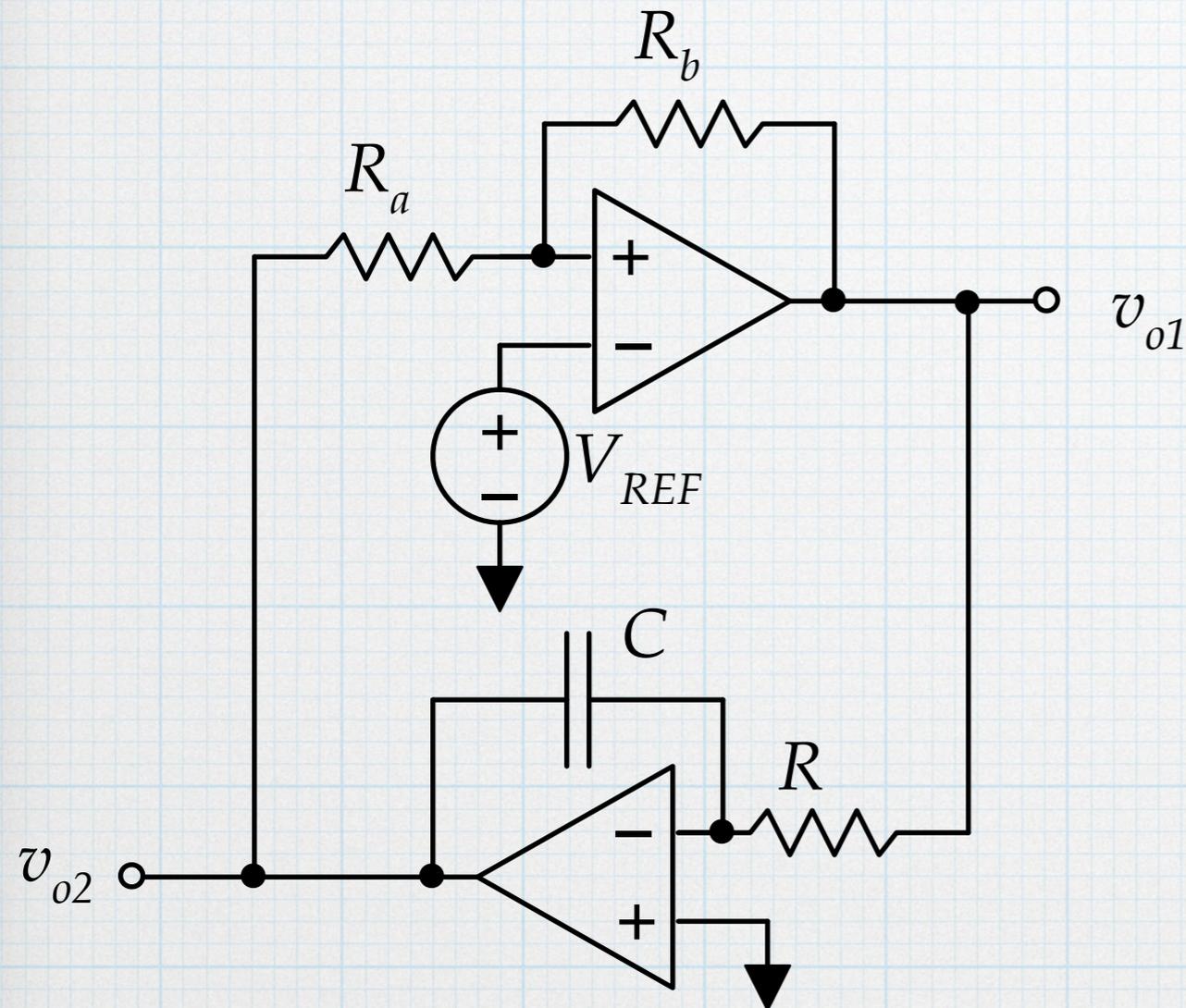
$$T_L = RC \ln\left(\frac{V_{S-} - V_{TH}}{V_{S-} - V_{TL}}\right)$$



Period:  $T = T_H + T_L = RC \ln \left[ \frac{(V_{S+} - V_{TL})(V_{S-} - V_{TH})}{(V_{S+} - V_{TH})(V_{S-} - V_{TL})} \right]$

# non-linear oscillator - version 2 (function generator)

A variation on the previous circuit gives another oscillator. In this case, a non-inverting comparator is used with an inverting integrator tying the output back to the input. Assume symmetric supplies:  $V_{L-} = -V_{L+}$



$$v_{o2}(t) = -\frac{1}{RC} \int_0^t v_{o1}(\tau) d\tau + v_{o2}(0)$$

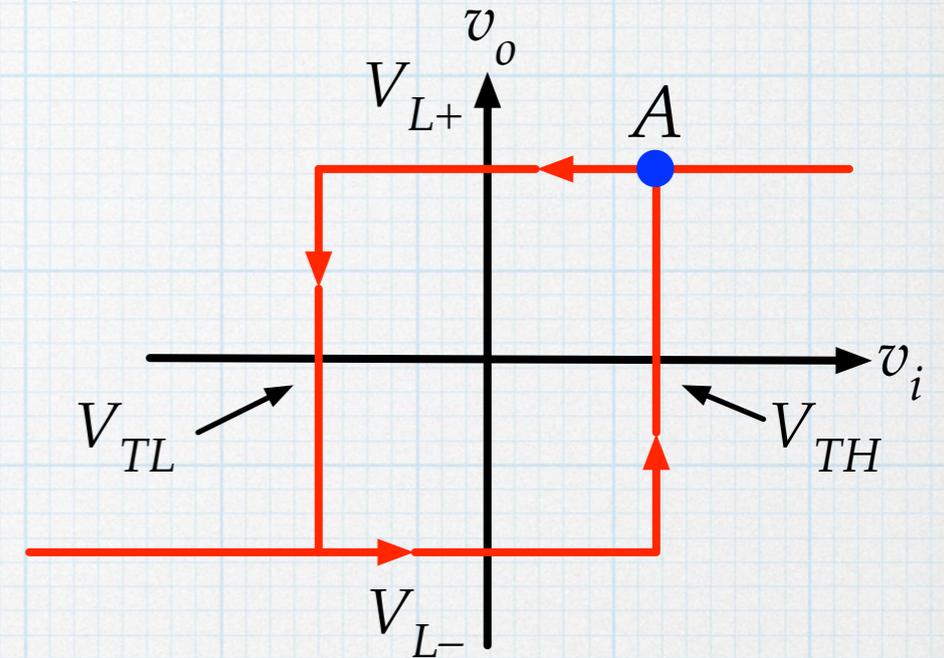
$$= -\frac{V_L}{RC} t + v_{o2}(0)$$

1. Start with the input to the comparator ( $v_{o2}$ ) high. Then the output is also high,  $v_{o1} = V_{L+}$ . Since the comparator output is also the integrator input, the constant positive voltage causes the integrator output to ramp down to lower voltage.
2. When the output of the integrator drops to  $V_{TL}$ , the comparator will switch to the low-output state,  $v_{o1} = V_{L-}$ .
3. The integrator output ramps *up* in voltage until it hits  $V_{TH}$ .
4. The comparator switches to the high output state, and the cycle repeats.

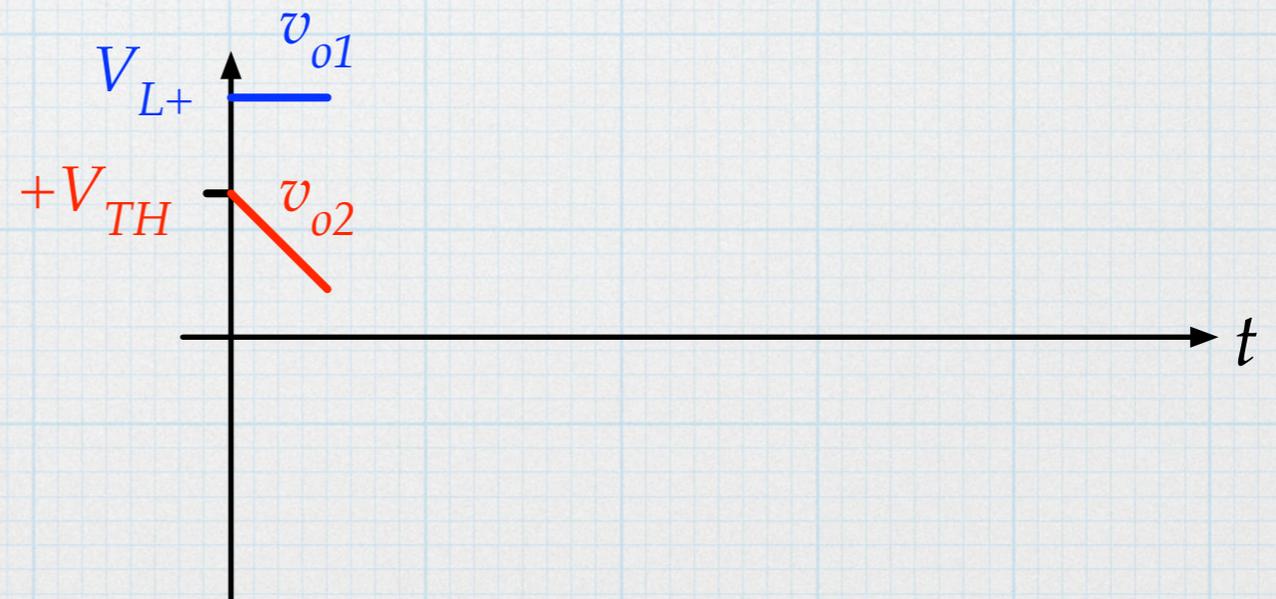
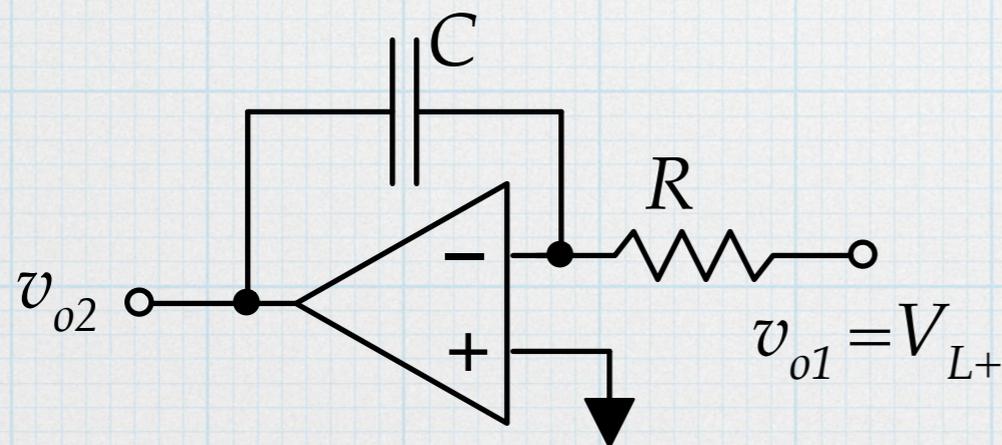
# Output high

The comparator will always be operating at some point on its transfer characteristic.

Start by assuming the output has just switched high – the comparator input (integrator output) has just reached  $V_{TH}$ . This is point A on the transfer characteristic. The output voltage is at  $V_{L+}$  and  $v_{o2} = V_{TH}$ .

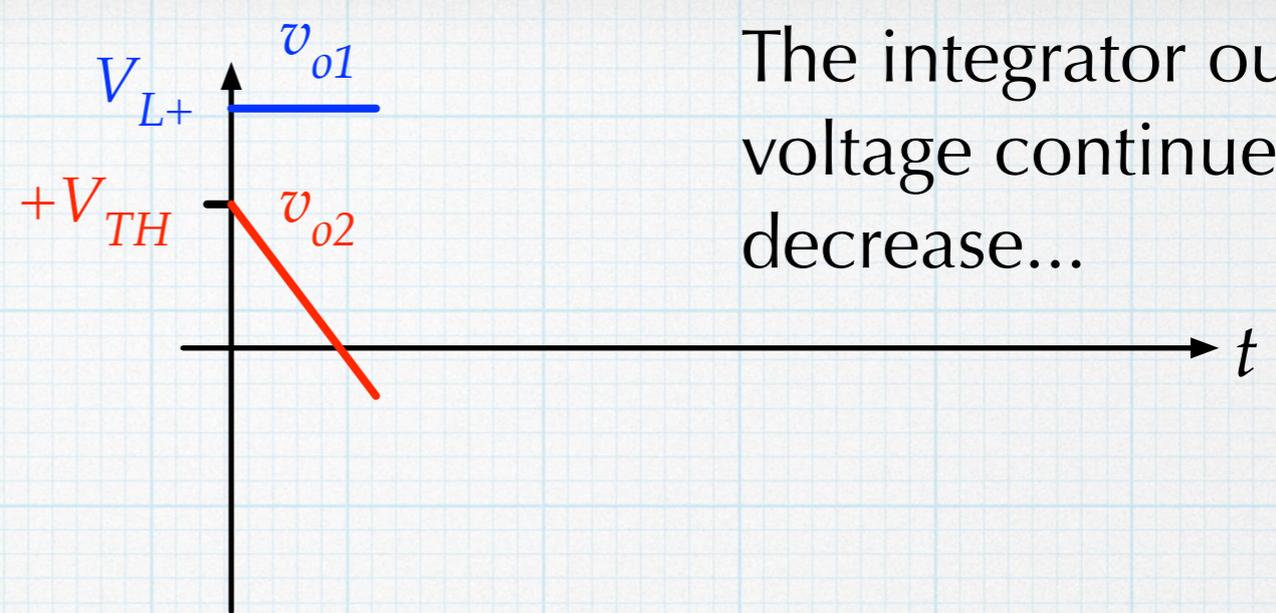


The input to the integrator is  $V_{L+}$ . This causes the integrator output to ramp down from  $V_{TH}$ .

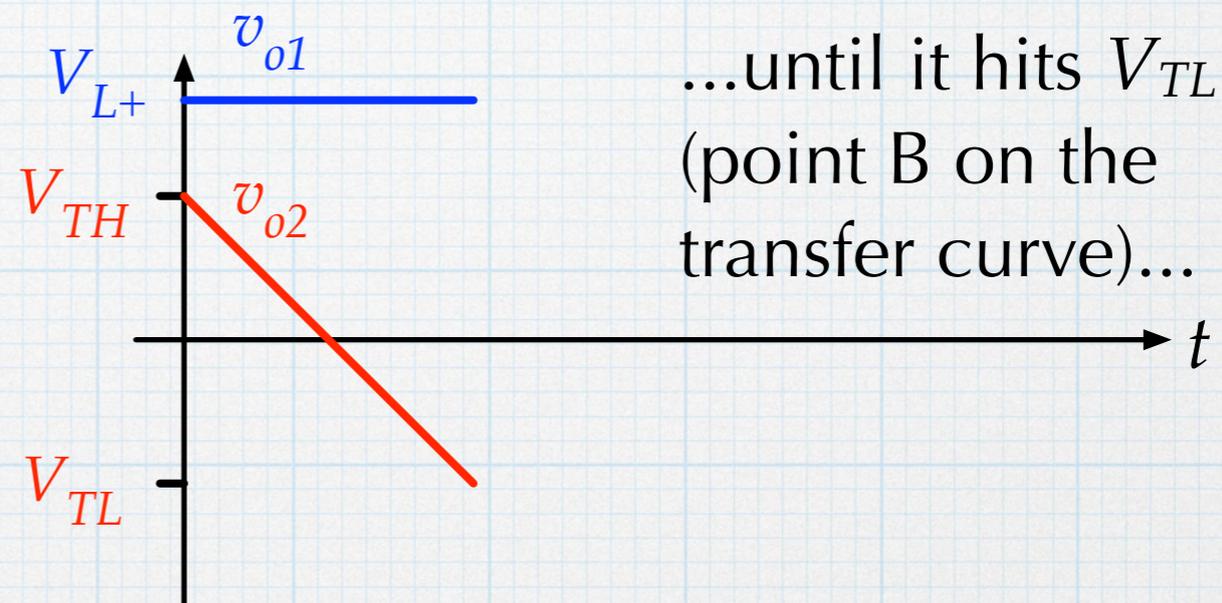


$$v_{o2}(t) = V_{TH} - \frac{V_{L+}}{RC}t$$

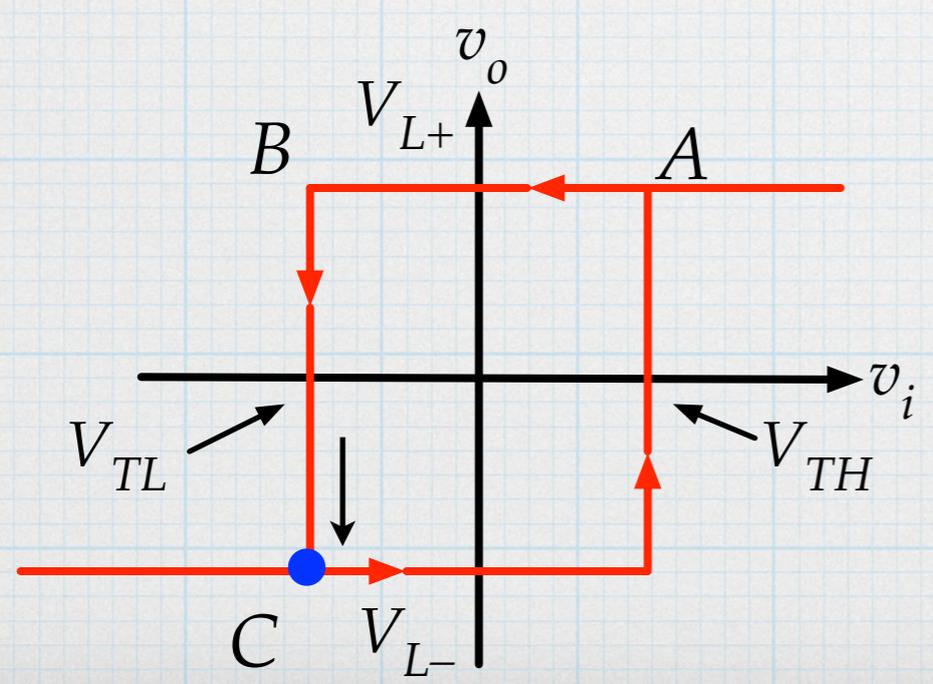
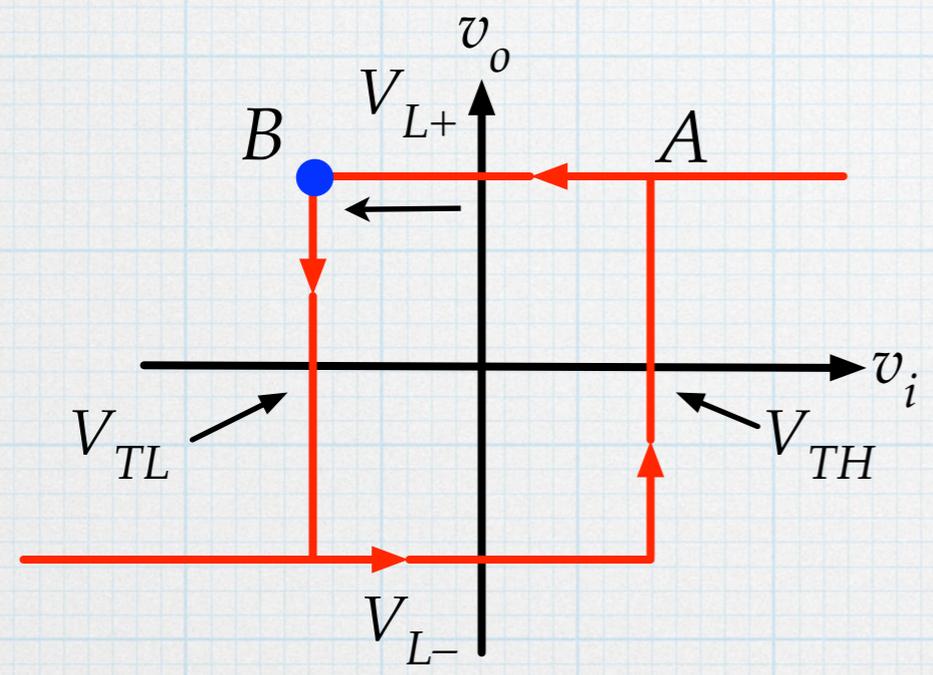
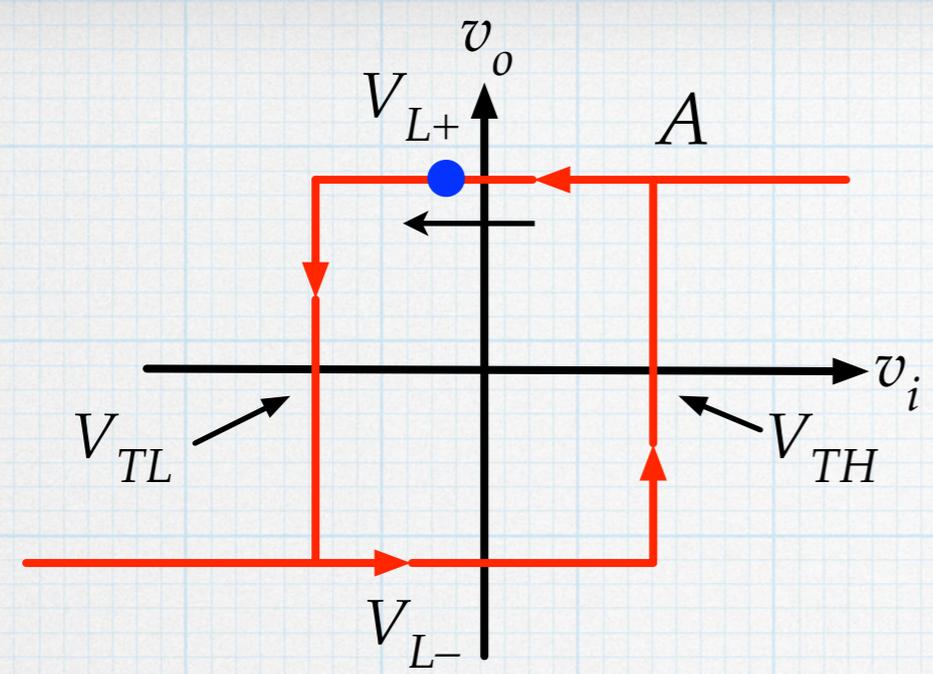
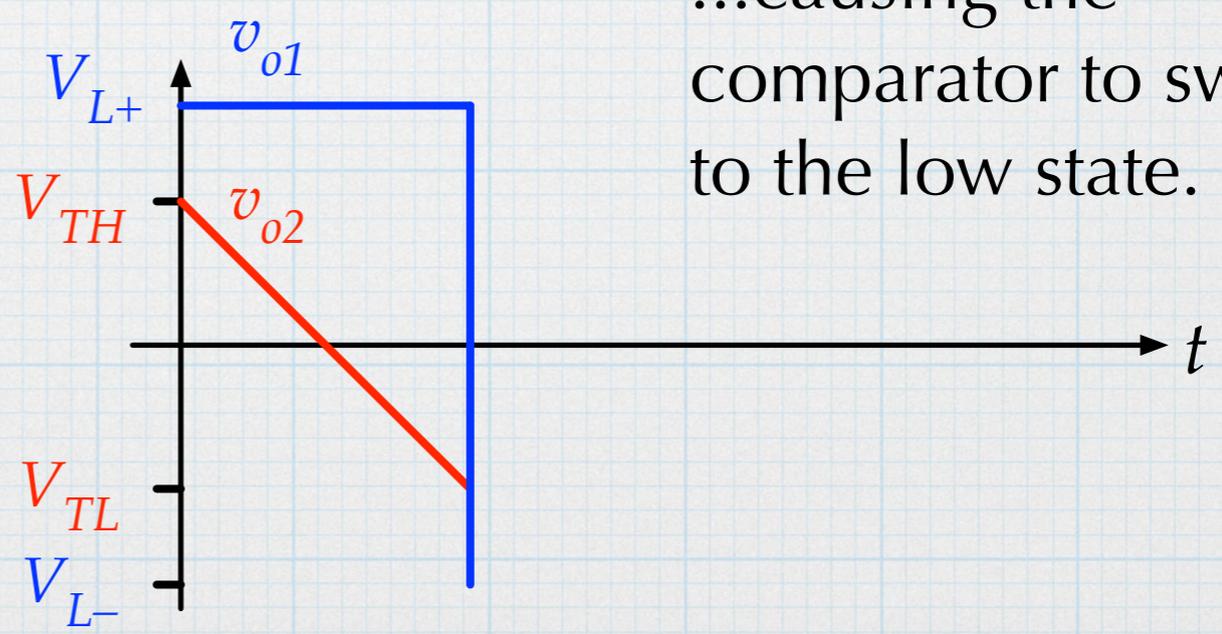
The integrator output voltage continues to decrease...



...until it hits  $V_{TL}$  (point B on the transfer curve)...



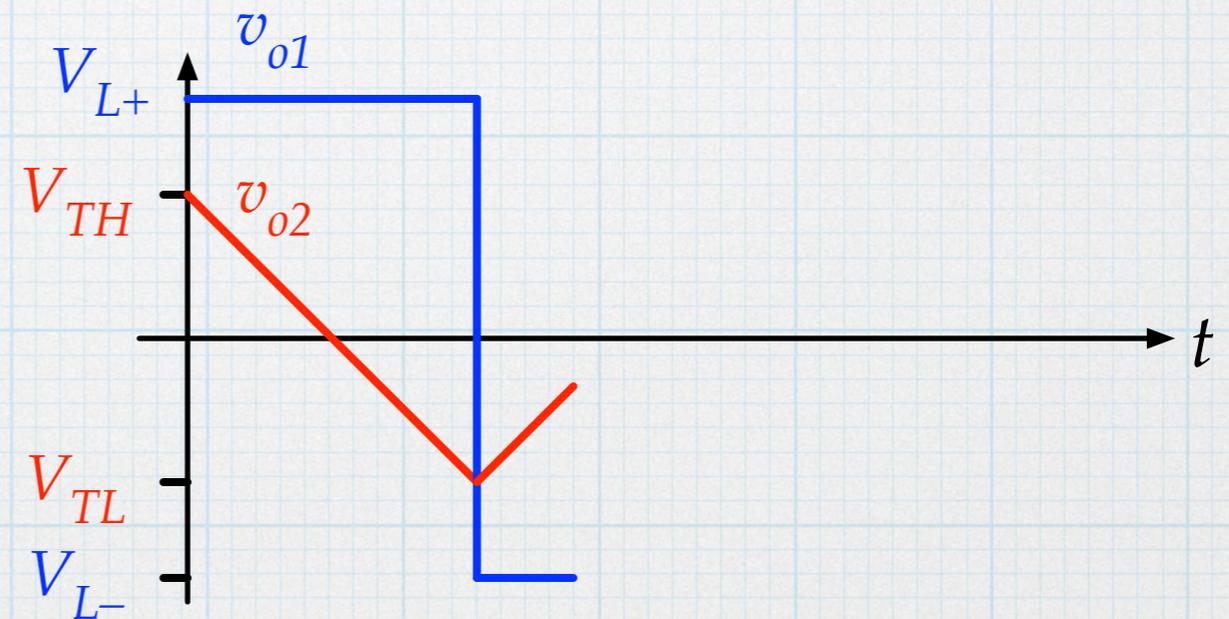
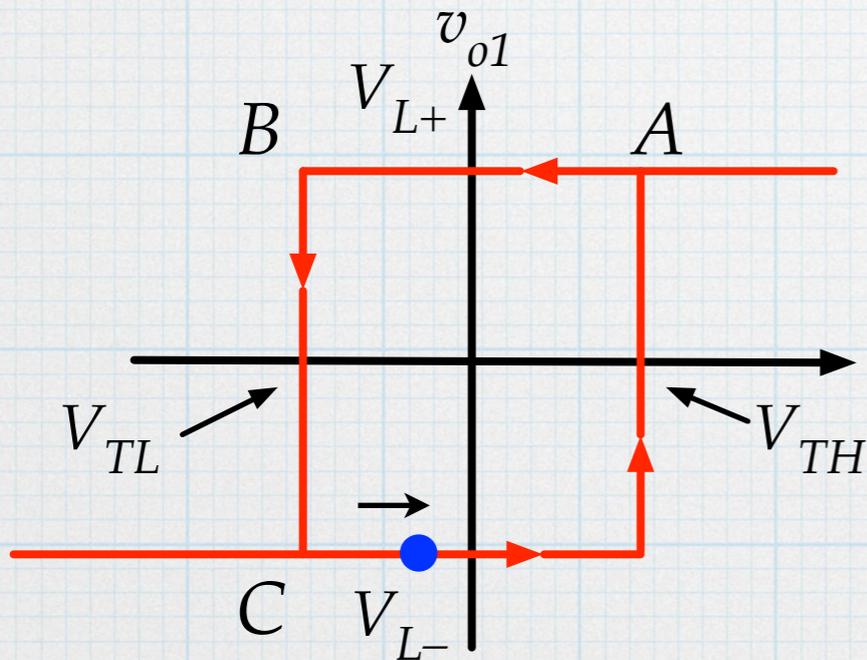
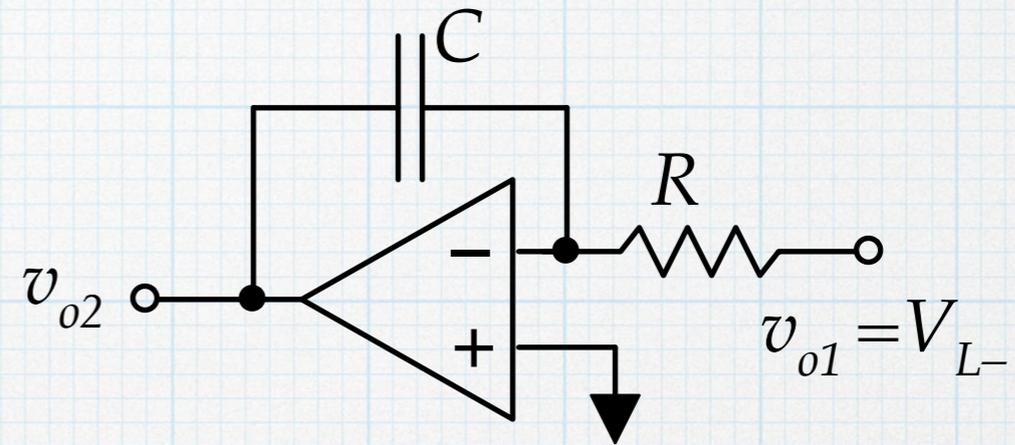
...causing the comparator to switch to the low state.

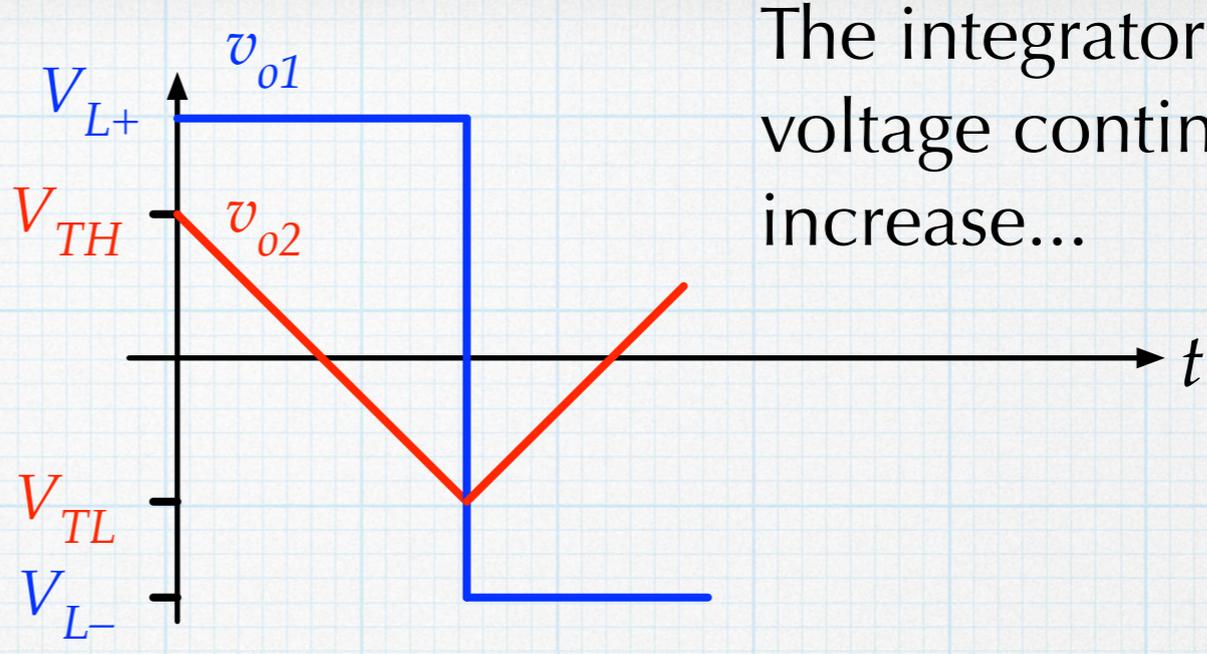
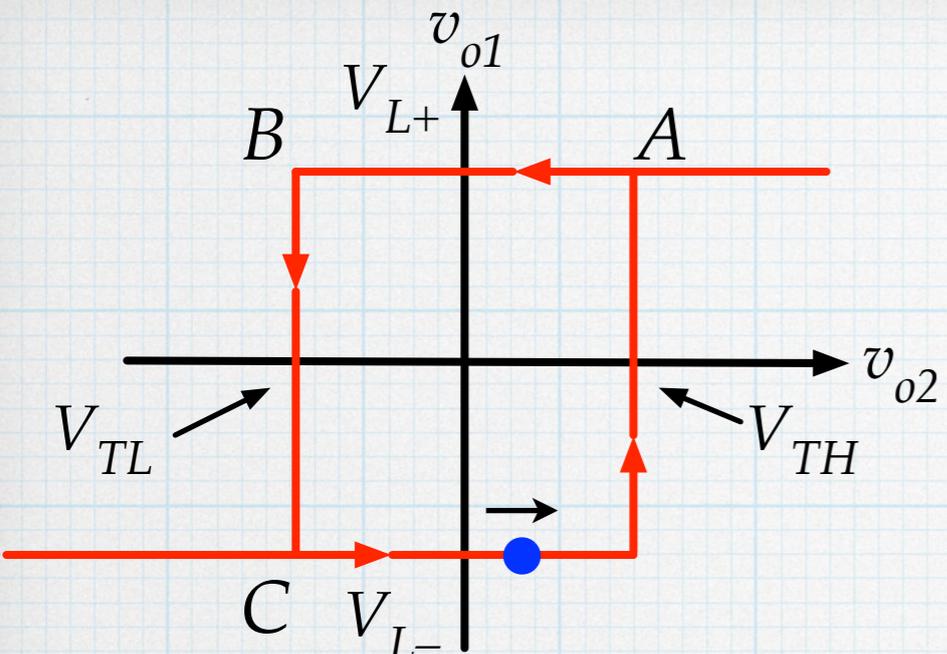


# Output low

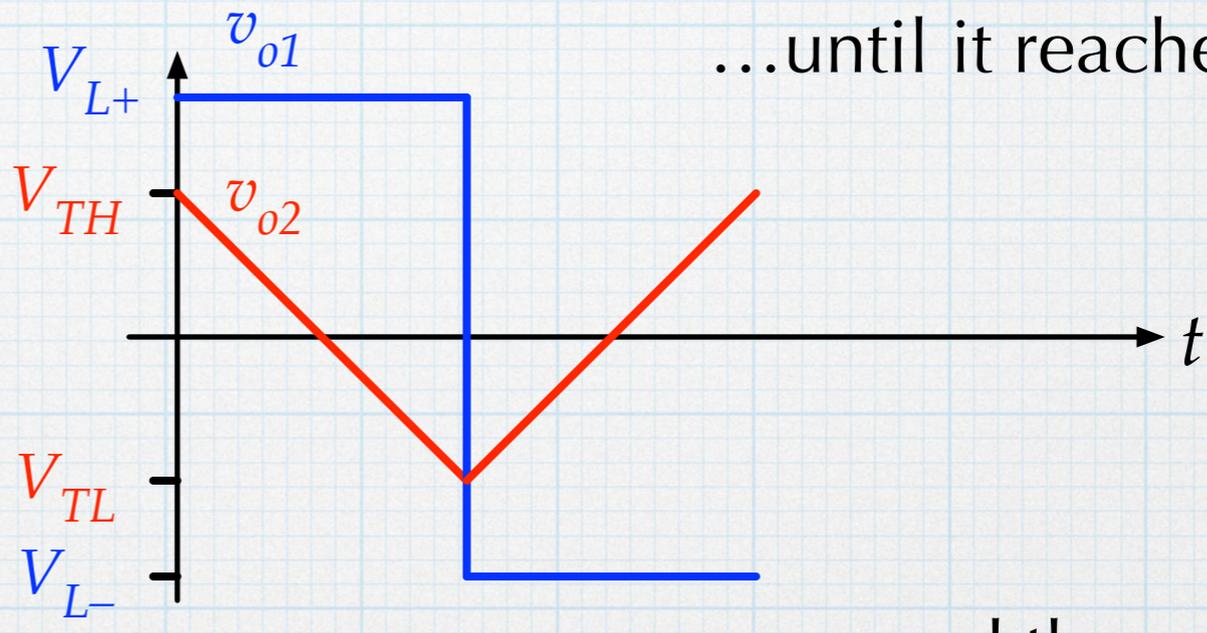
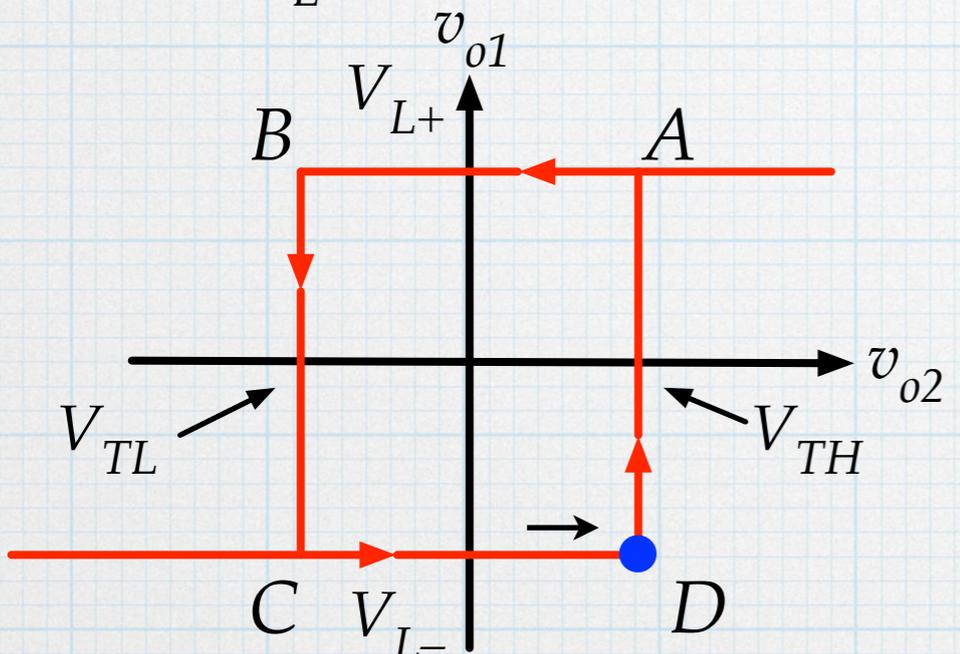
Now the comparator is at point C on the transfer characteristic, with  $v_{o1} = V_{L-}$  (recall that  $V_{L-}$  is negative) and  $v_{o2} = V_{TL}$ . The integrator output voltage will start to ramp up.

$$v_{o2}(t) = V_{TL} + \frac{|V_{L-}|}{RC}t$$

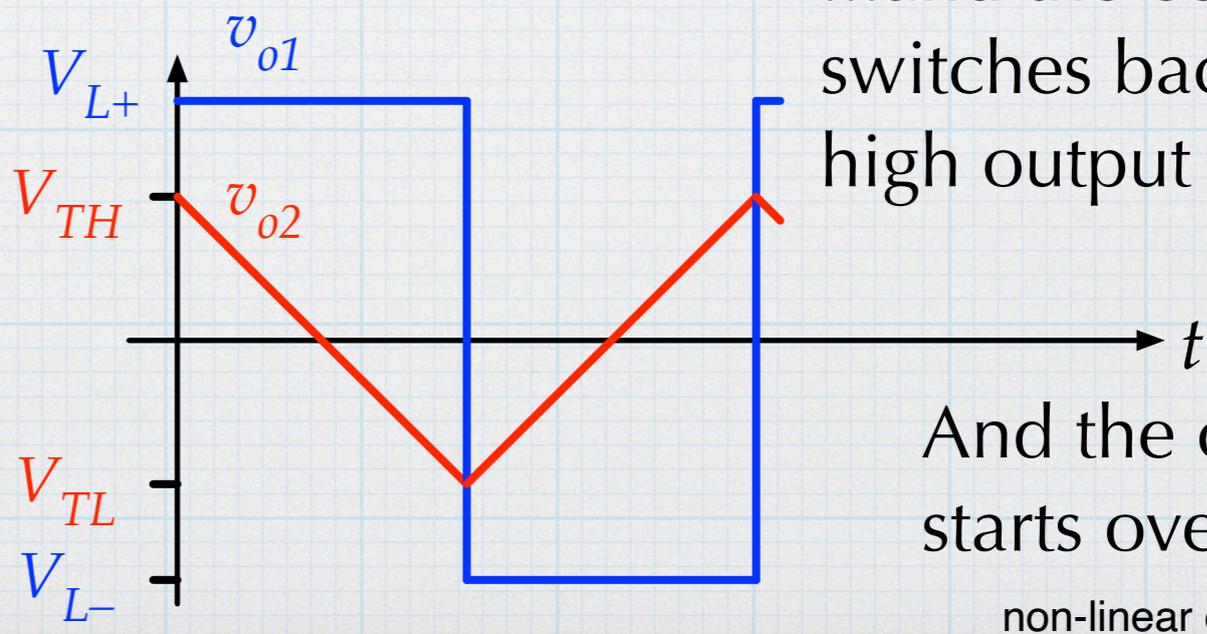
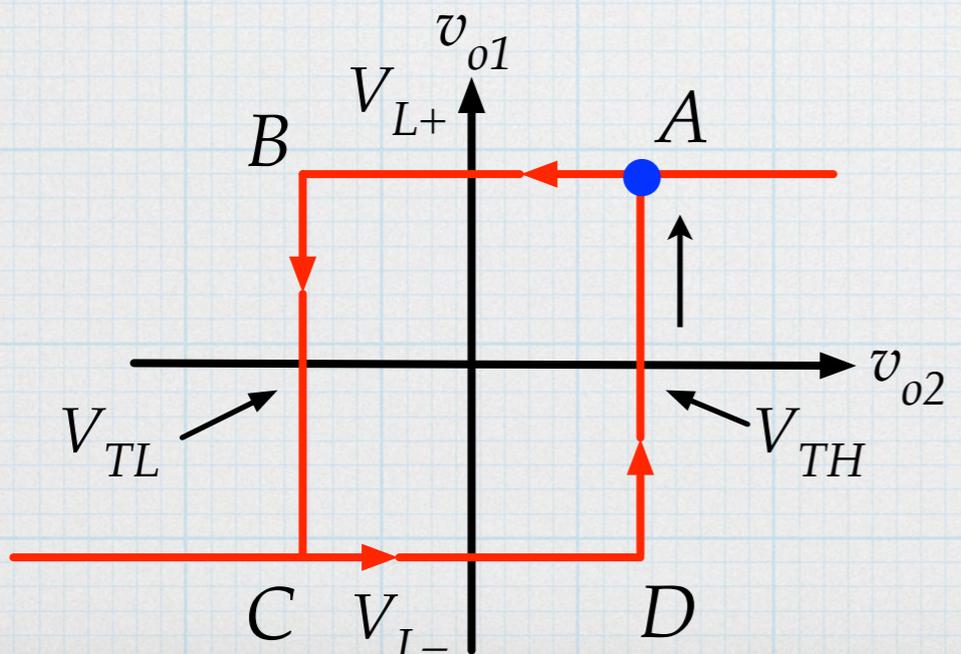




The integrator output voltage continues to increase...



...until it reaches  $V_{TH}$ ...

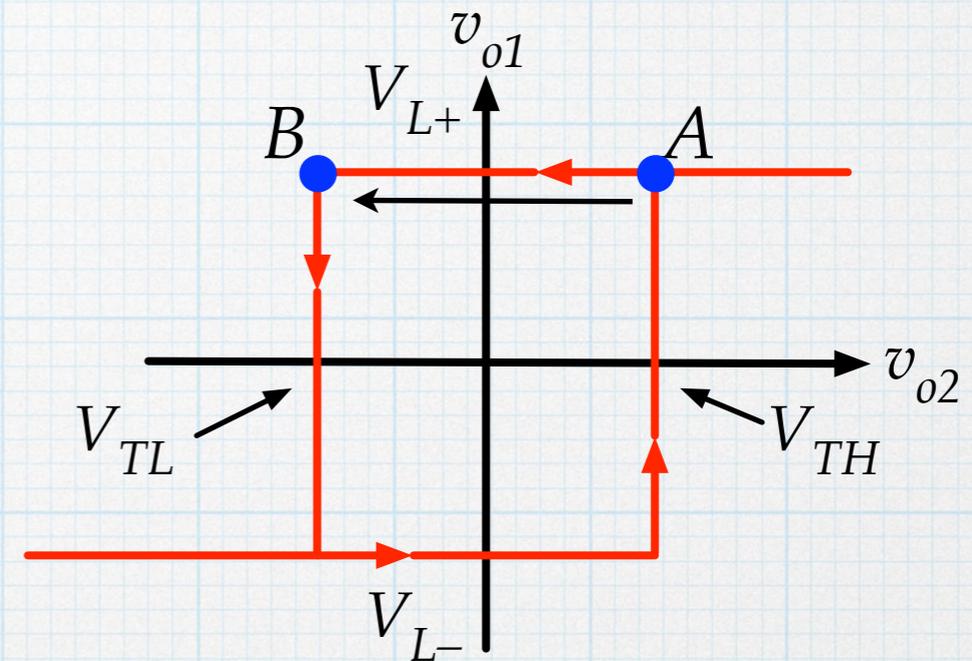


...and the comparator switches back to the high output state.

And the cycle starts over.

The total period of the oscillation will be determined by the time needed for the integrator to ramp up and down between  $V_{TL}$  and  $V_{TH}$ . This is represented by the times needed to move across the top and bottom of the transfer characteristic.

The time that the comparator output is high is represented by the transition from  $A$  to  $B$  on the transfer characteristic. Use the integrator equation with  $v_{o1} = V_{L+}$ .



$$v_{o2}(t) = V_{TH} - \frac{V_{L+}}{RC}t$$

$$V_{TL} = V_{TH} - \frac{V_{L+}}{RC}T_H$$

$$T_H = RC \left[ \frac{V_{TH} - V_{TL}}{V_{L+}} \right] = RC \left[ \frac{\Delta V_T}{V_{L+}} \right]$$

The time that the comparator output is low is represented by the transition from  $C$  to  $D$  on the transfer characteristic. Use the integrator equation with  $v_{o1} = V_{L-}$ .

$$v_{o2}(t) = V_{TL} + \frac{|V_{L-}|}{RC}t$$

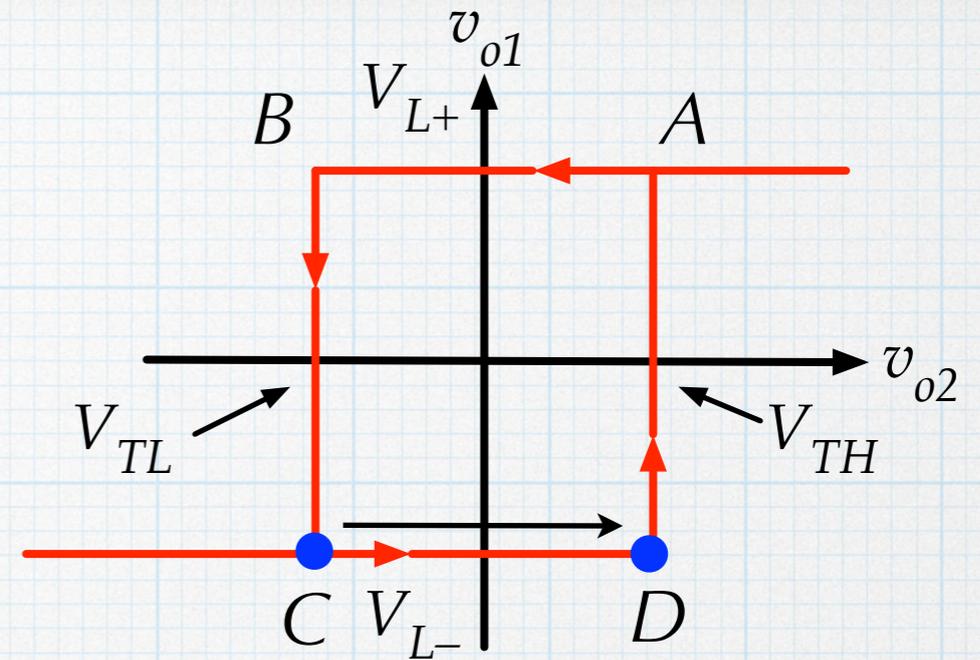
$$V_{TH} = V_{TL} + \frac{|V_{L-}|}{RC}T_L$$

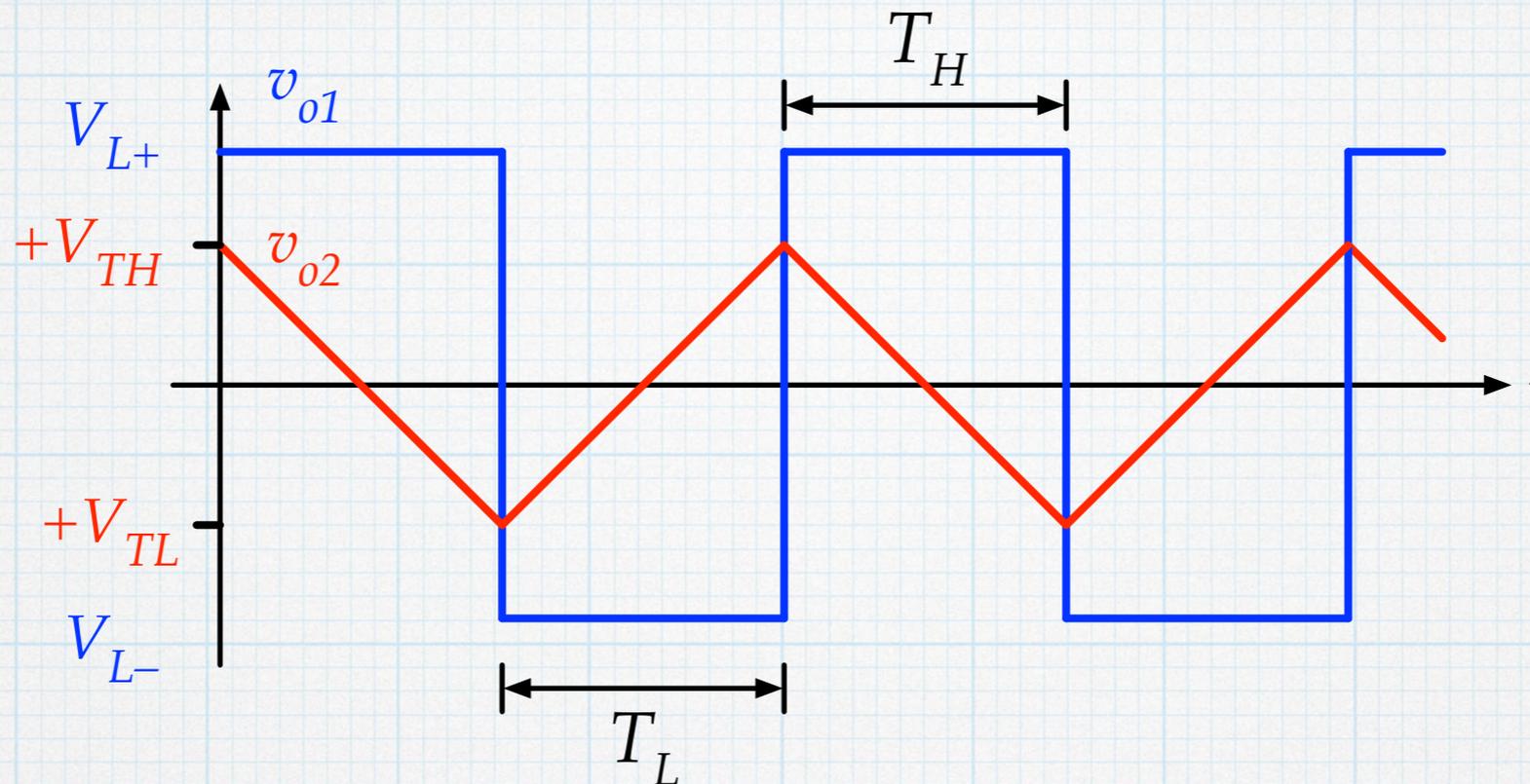
$$T_L = RC \left[ \frac{V_{TH} - V_{TL}}{|V_{L-}|} \right] = RC \left[ \frac{\Delta V_T}{|V_{L-}|} \right]$$

$$T = T_H + T_L = RC (\Delta V_T) \left[ \frac{1}{V_{L+}} + \frac{1}{|V_{L-}|} \right]$$

If  $V_{L-} = -V_{L+}$  (power supply voltages are symmetric)

$$T = 2RC \left[ \frac{\Delta V_T}{V_L} \right]$$





The output of the comparator will be a square wave, with high and low voltages given by the output limits and high and low times determined by the specifics of the hysteresis curve and the RC time constant of the integrator.

The output of the integrator will be a triangle or ramp wave, ramping up and down between  $V_{TL}$  and  $V_{TH}$ , with the same period as the square wave.

As an added bonus, it is relatively easy to convert the triangle to a sine wave using a filter that attenuates the higher order harmonics.