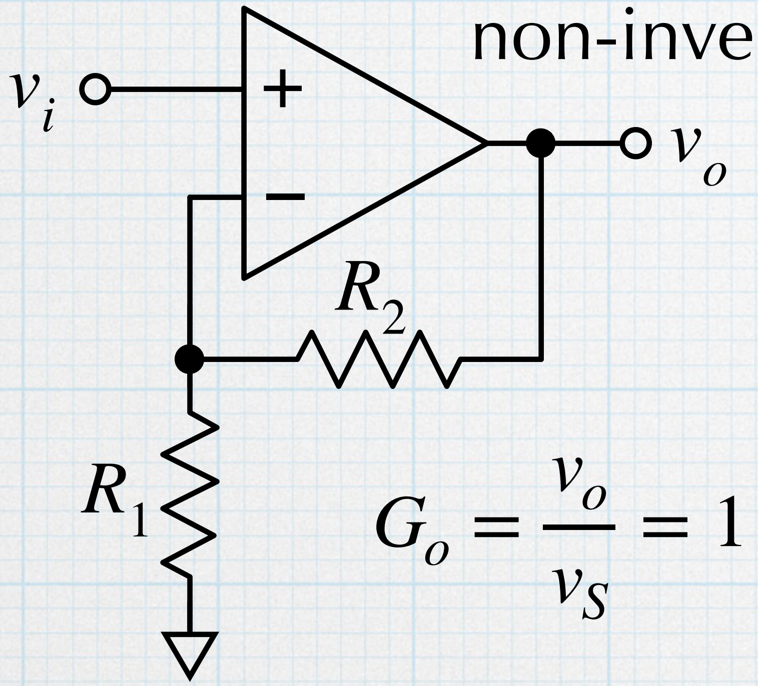
More details about ideal op amps

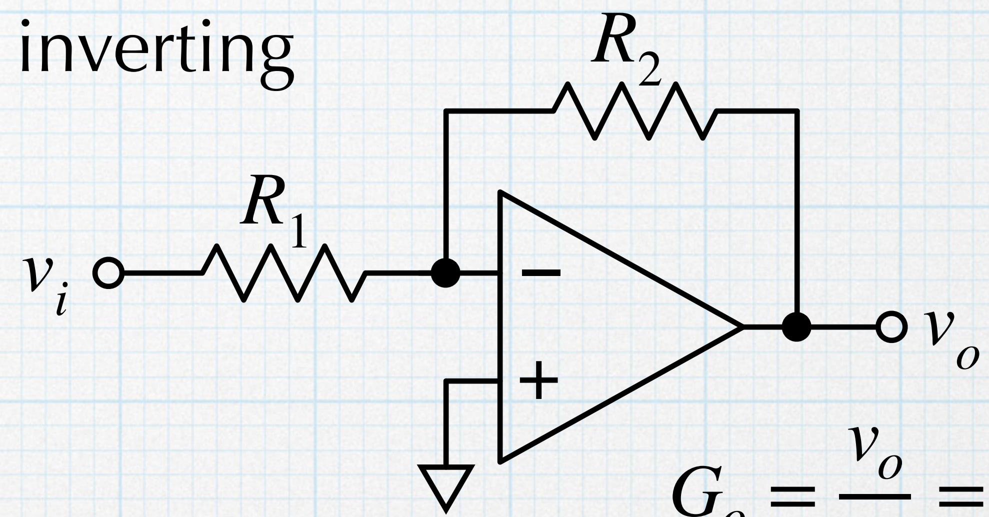
We had our first look at op amps and amplifier circuits in EE 201. In EE 230, op amps play a central in many of the electronics applications we want to study. Let's dig deeper on ideal amps. Recall the 4 basic circuits:

non-inverting



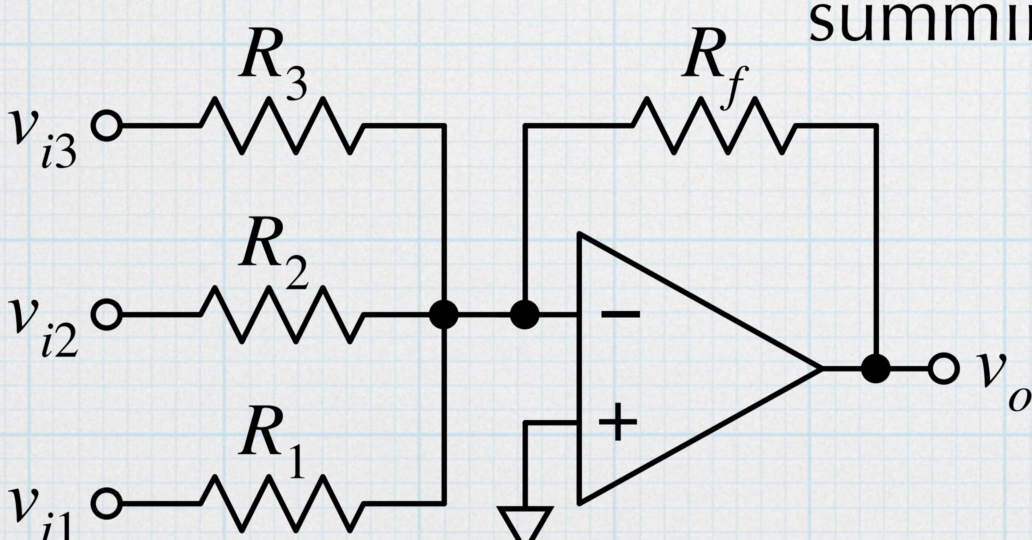
$$G_o = \frac{v_o}{v_S} = 1 + \frac{R_2}{R_1}$$

inverting



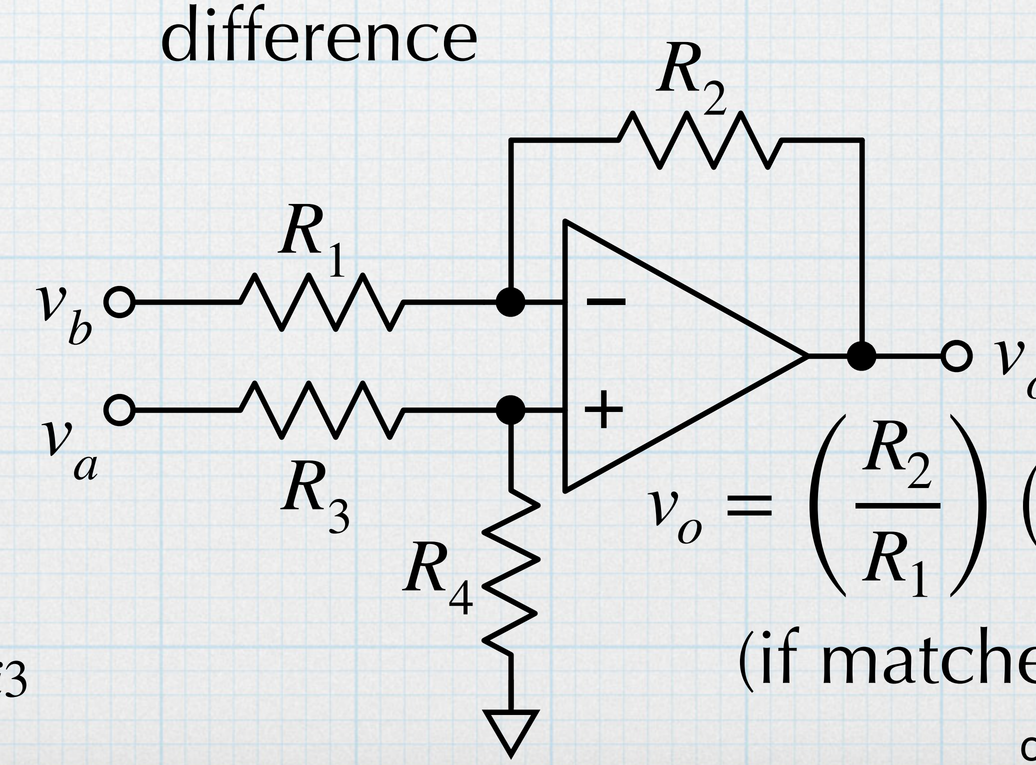
$$G_o = \frac{v_o}{v_i} = -\frac{R_2}{R_1}$$

summing



$$v_o = -\frac{R_f}{R_1}v_{i1} - \frac{R_f}{R_2}v_{i2} - \frac{R_f}{R_3}v_{i3}$$

difference



$$v_o = \left(\frac{R_2}{R_1}\right)(v_a - v_b)$$

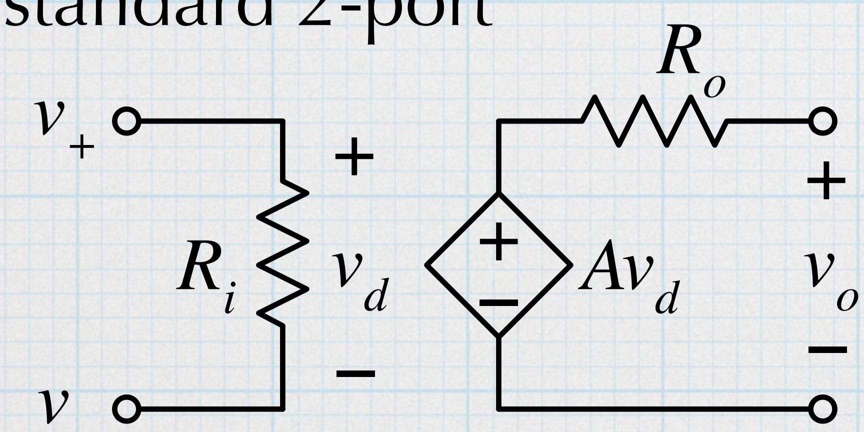
(if matched)

Power supplies and ground

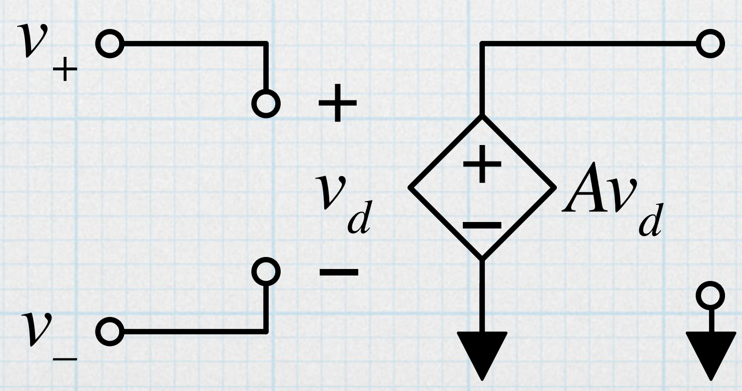
As we have seen previously, an amp needs one or more DC supplies in order to function. The extra power that is added to the signal when it is amplified comes from the DC supplies. Without them, the amp is just a dead element squatting in the middle of the circuit. We don't always show the supplies in the circuit diagram, but they must be present in the real circuit. The common connection for the supplies becomes a convenient node to define as the ground in the circuit. Since all electronic circuits must have power supplies, the circuit ground will (almost) always be defined by one of the terminals of a DC source. This ground allows for some "short-hand" in the circuit diagrams.

Also, the output of a standard op amp is referenced to the power supply ground. (There are differential output amps, but they are specialty components.)

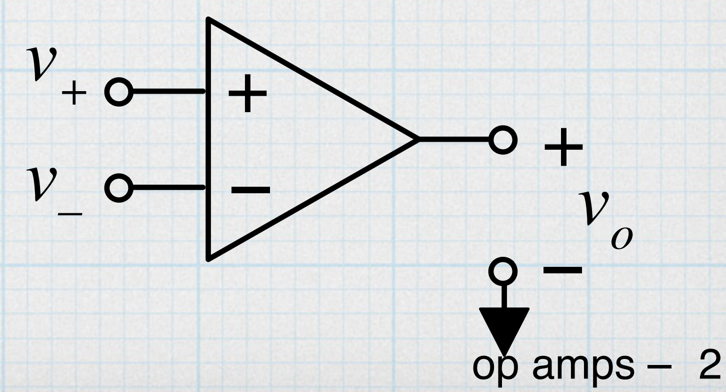
standard 2-port



ideal, with ground

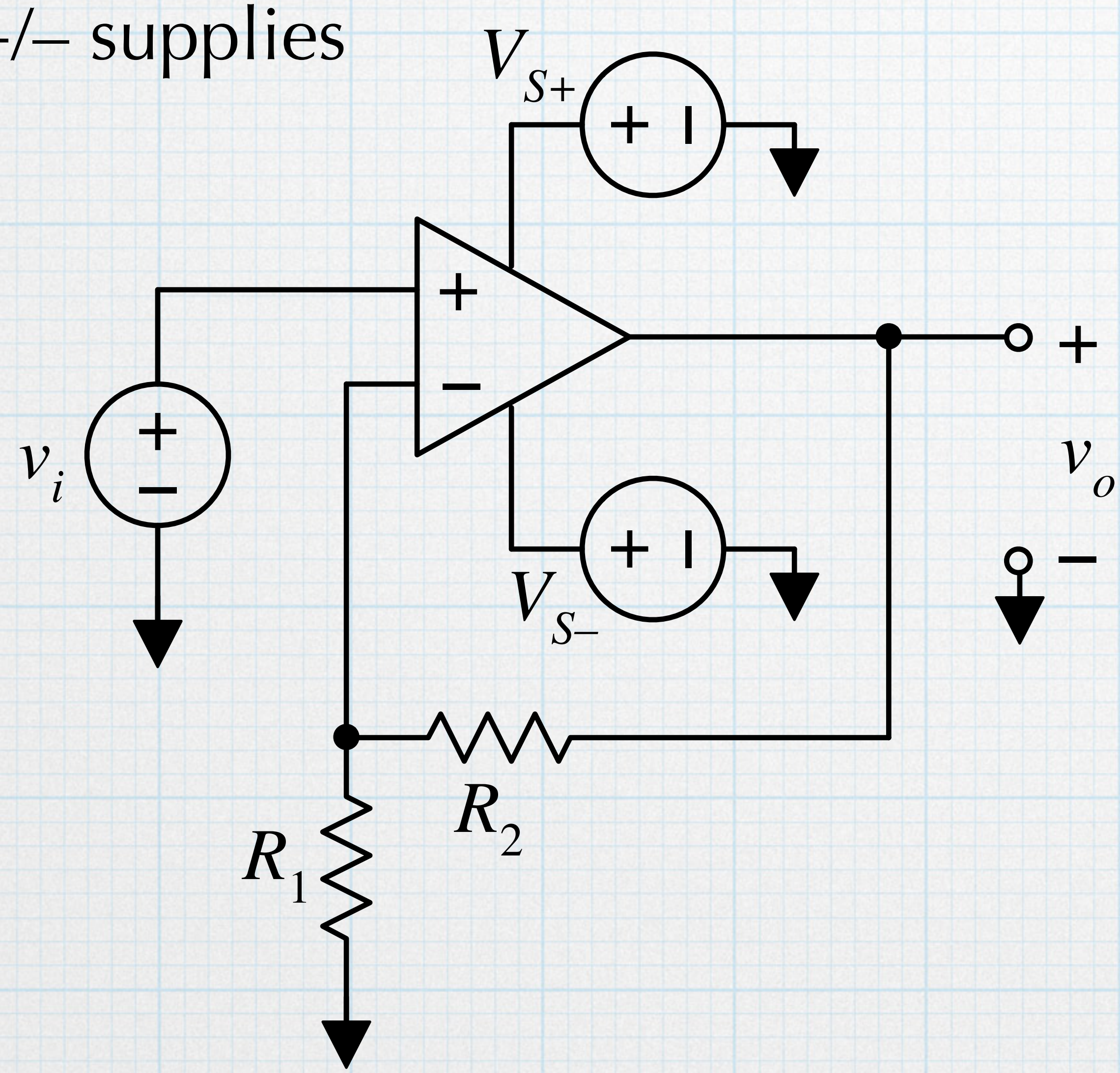


generic ideal op amp

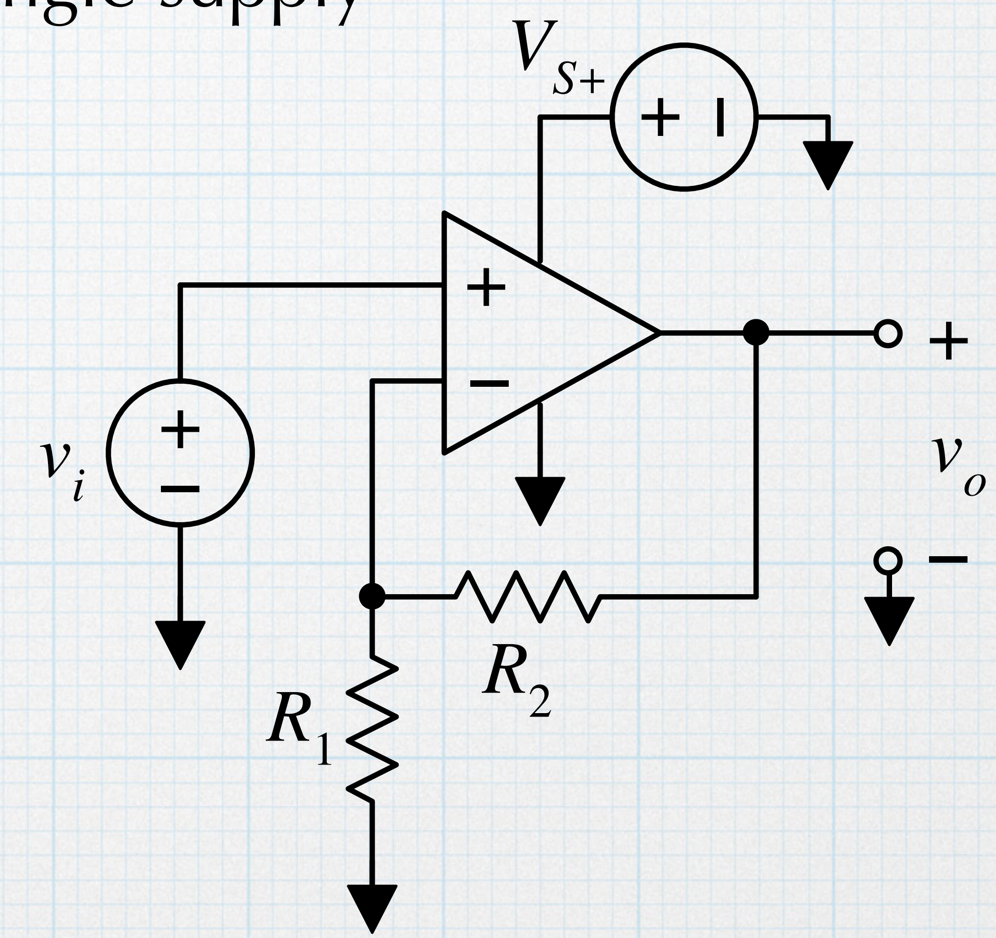


We could include all of the power supplies and other sources explicitly. Then amp with voltage-divider feedback might look like this:

+/– supplies



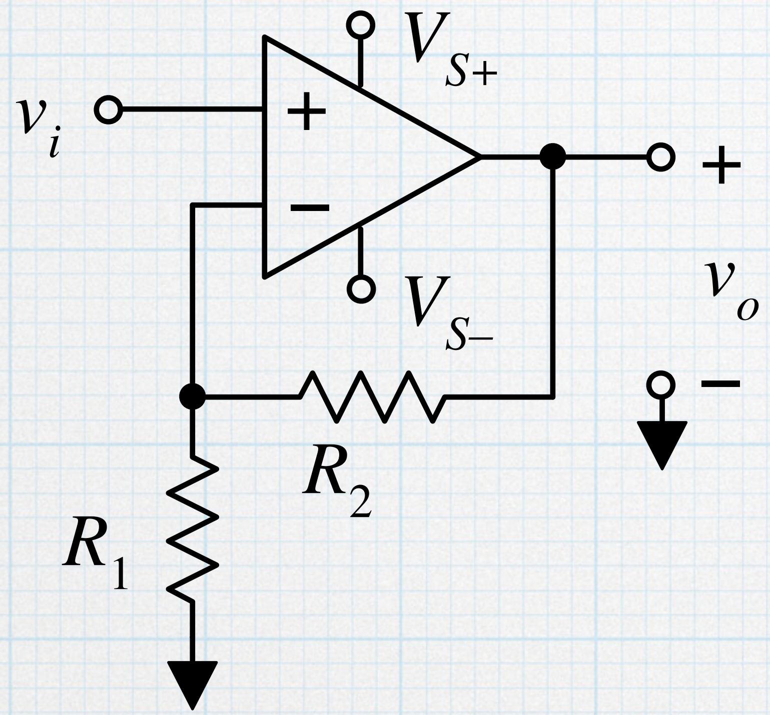
single supply



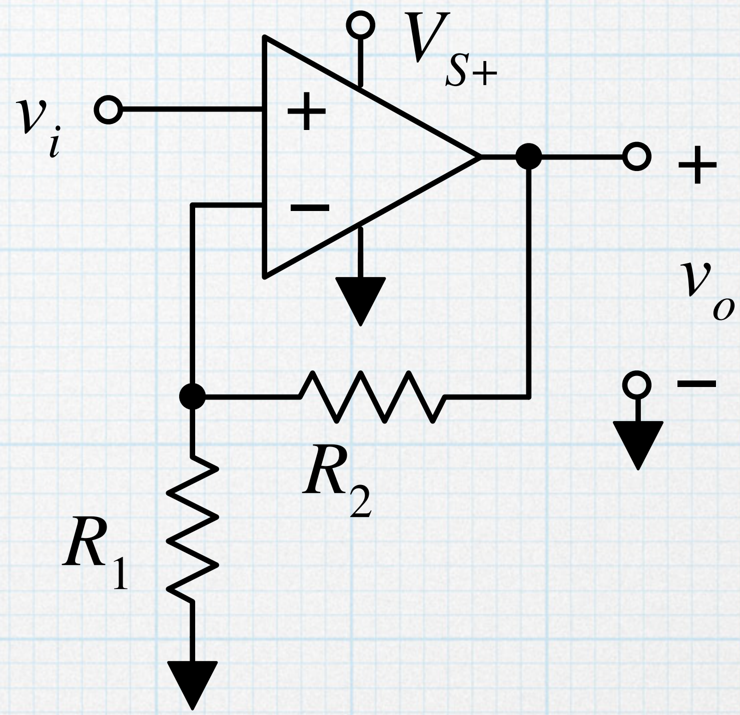
These are kind of messy.

We rarely draw in the power supplies explicitly. (Except maybe in SPICE.) To simplify, we might indicate the presence of the sources (referenced to ground) by labeling the nodes where they are attached.

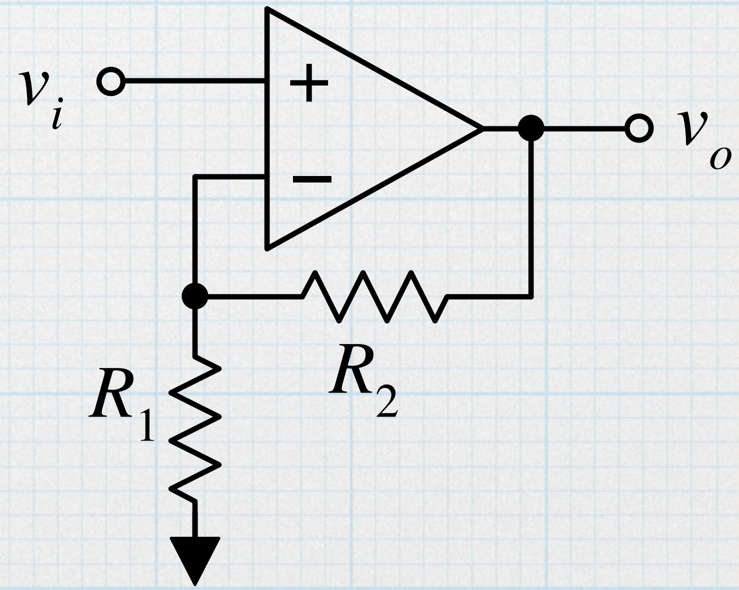
+/– supplies



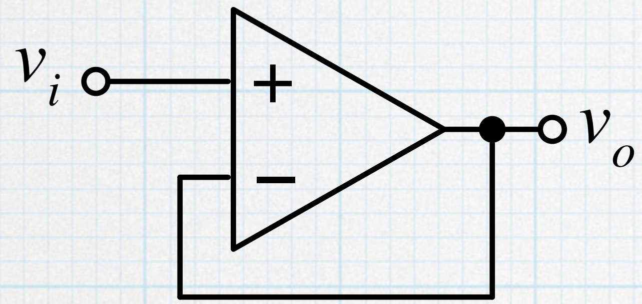
single supply



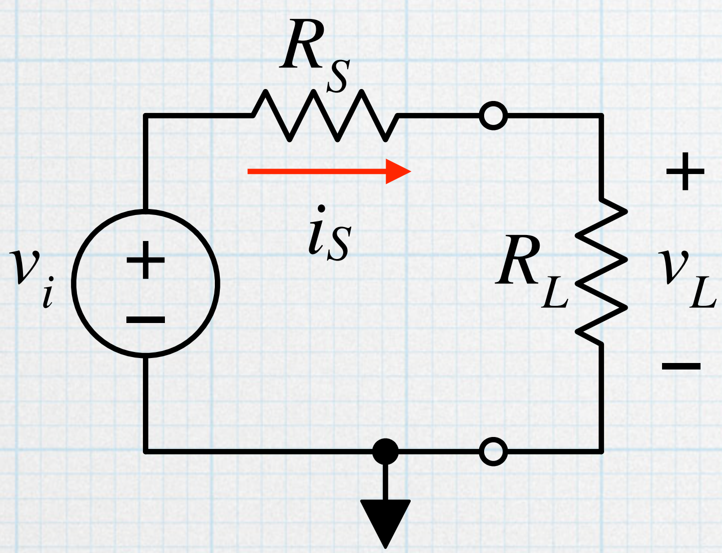
And most often, we ignore even the power supply connections, simplifying the circuit diagram further. (However, we must be aware of the power supplies being used, because they will affect the performance of the op amp.)



Unity gain buffer



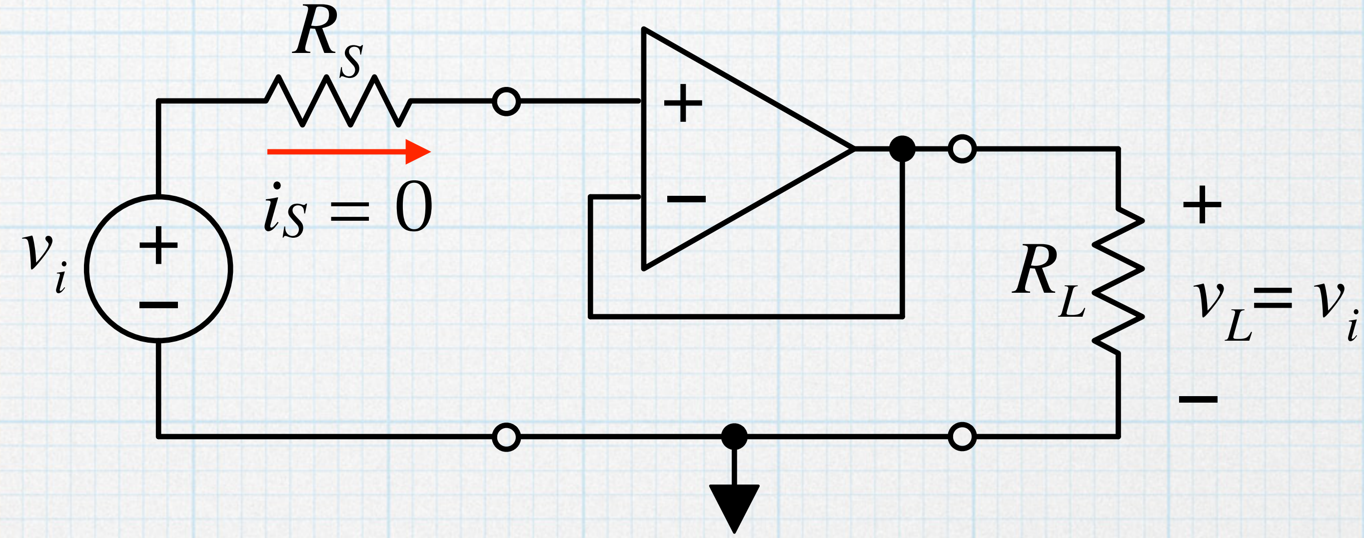
Non-inverting amp with $R_2 = 0$ and $R_1 \rightarrow \infty$. So $G = 1$, meaning $v_o = v_i$. This seems a bit dumb. What good is it?



Simple voltage divider.

$$v_L = \frac{R_L}{R_L + R_S} v_i$$

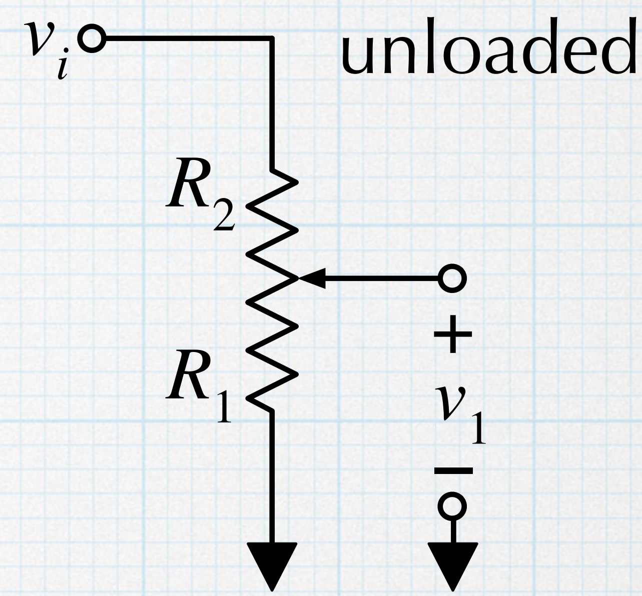
Unless $R_L >> R_S$, much of the voltage and power are lost in R_S.



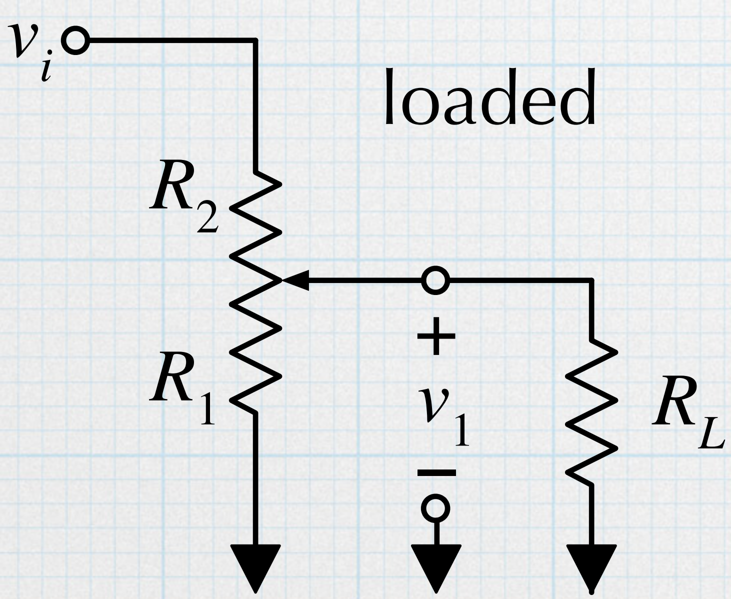
Insert the buffer. Now, $i_S = 0$ and $v_i = v_+ = v_- = v_L$. No voltage divider! The infinite input resistance and zero output resistance of the amp make this work. We can use the buffer anytime that we need to isolate two mismatched circuits.

A practical example

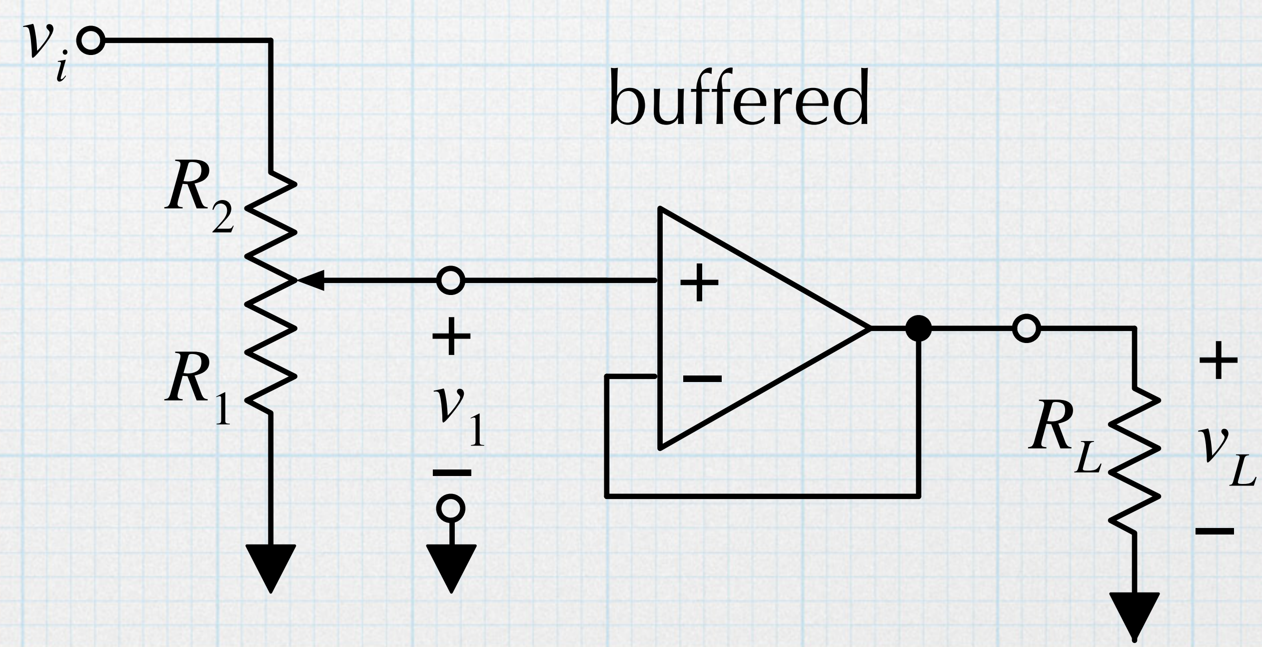
We might like to use a potentiometer as voltage divider to reduce a voltage level, as in the case of making a volume control for an audio amp or to provide a reference voltage from a power supply. However, as soon as a load is attached to the divider, the voltage changes. Recall that the total resistance of the potentiometer is divided into two resistances by the wiper, $R_T = R_1 + R_2$.



$$v_1 = \frac{R_1}{R_T} v_i$$



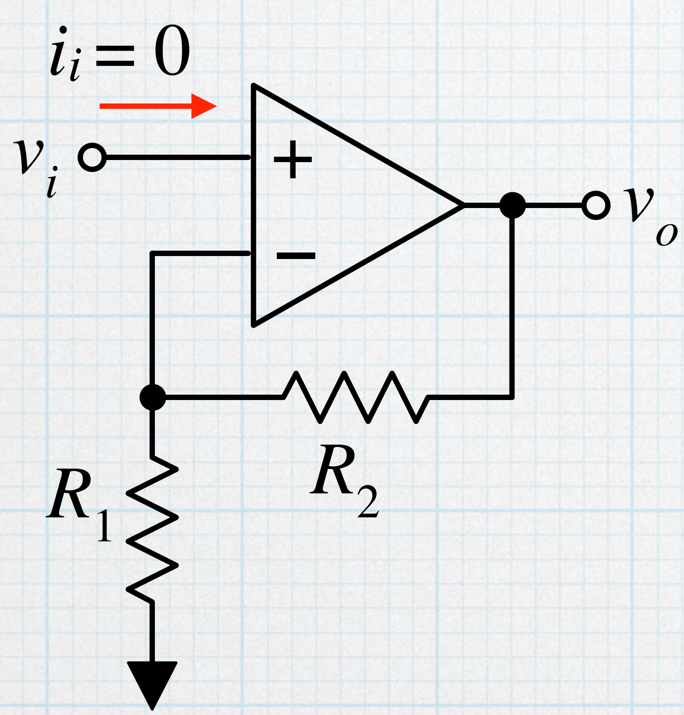
$$v_1 = \frac{R_1 \| R_L}{R_1 \| R_L + R_2} v_i$$



$$v_L = v_1 = \frac{R_1}{R_T} v_i$$

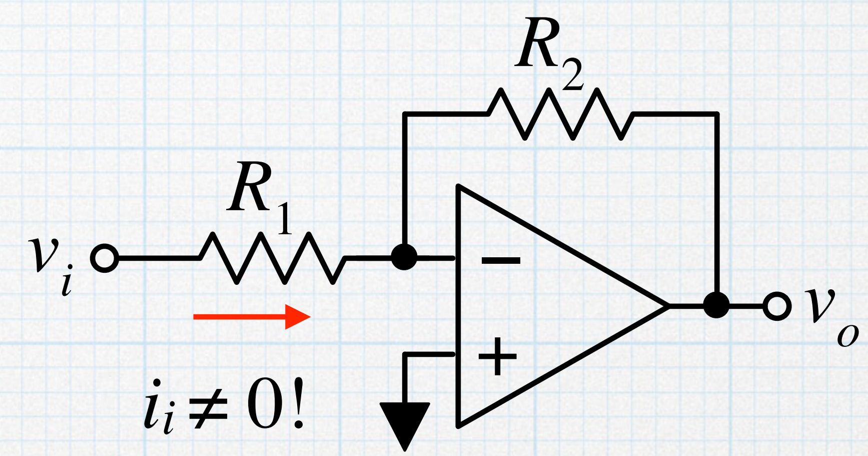
Watch out for the input resistance (and other changes)

It is important to note that even though the op amp is ideal, the combination of the op amp and the feedback network may not be ideal. As a case in point, consider the input resistance seen by the source in the non-inverting and inverting amps.



$R_i \to \infty$

Using the op amp to best effect.



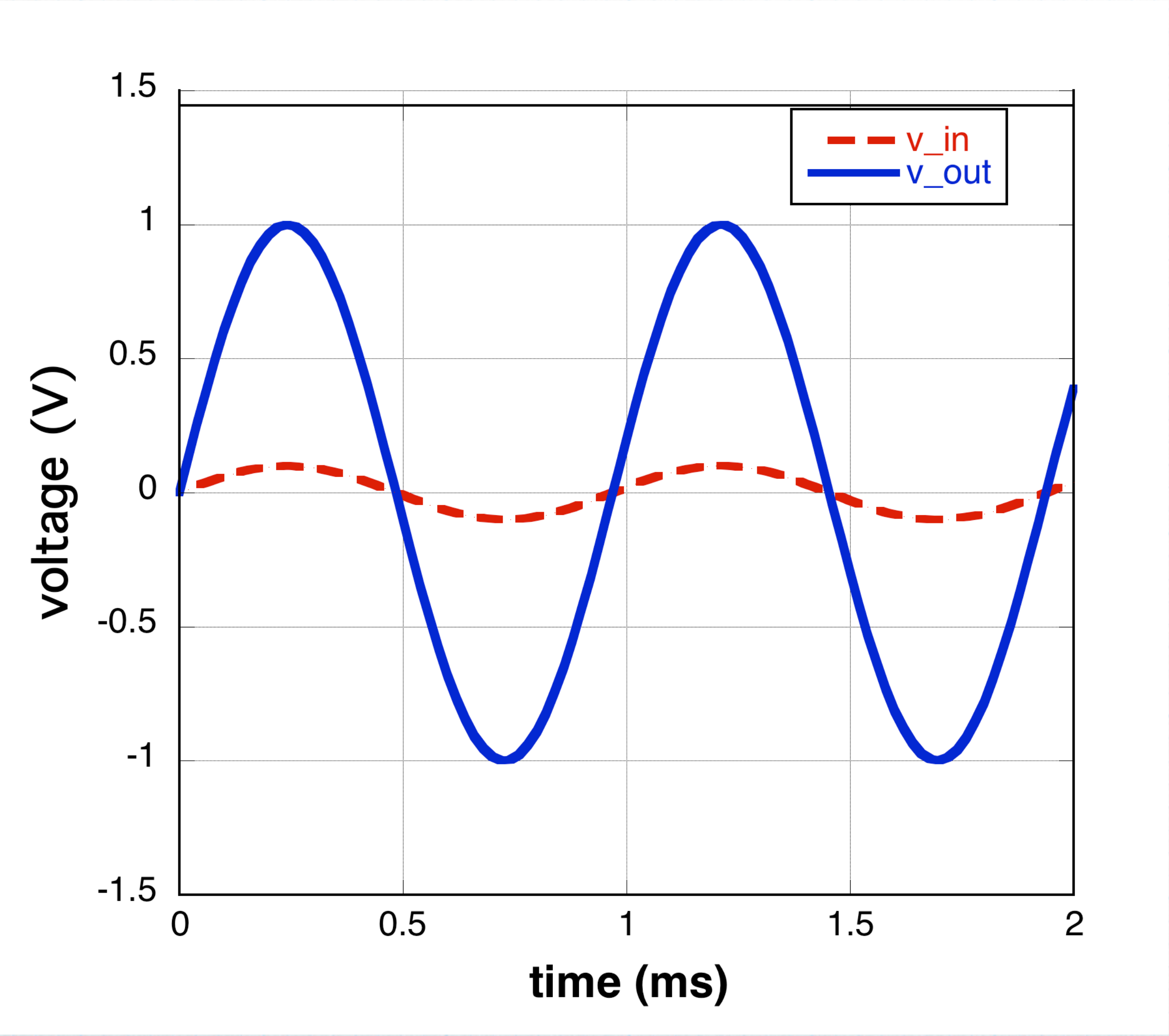
$$i_i = \frac{v_i - v_-}{R_1} = \frac{v_i}{R_1}$$

$$R_i = \frac{v_i}{i_i} = R_1$$

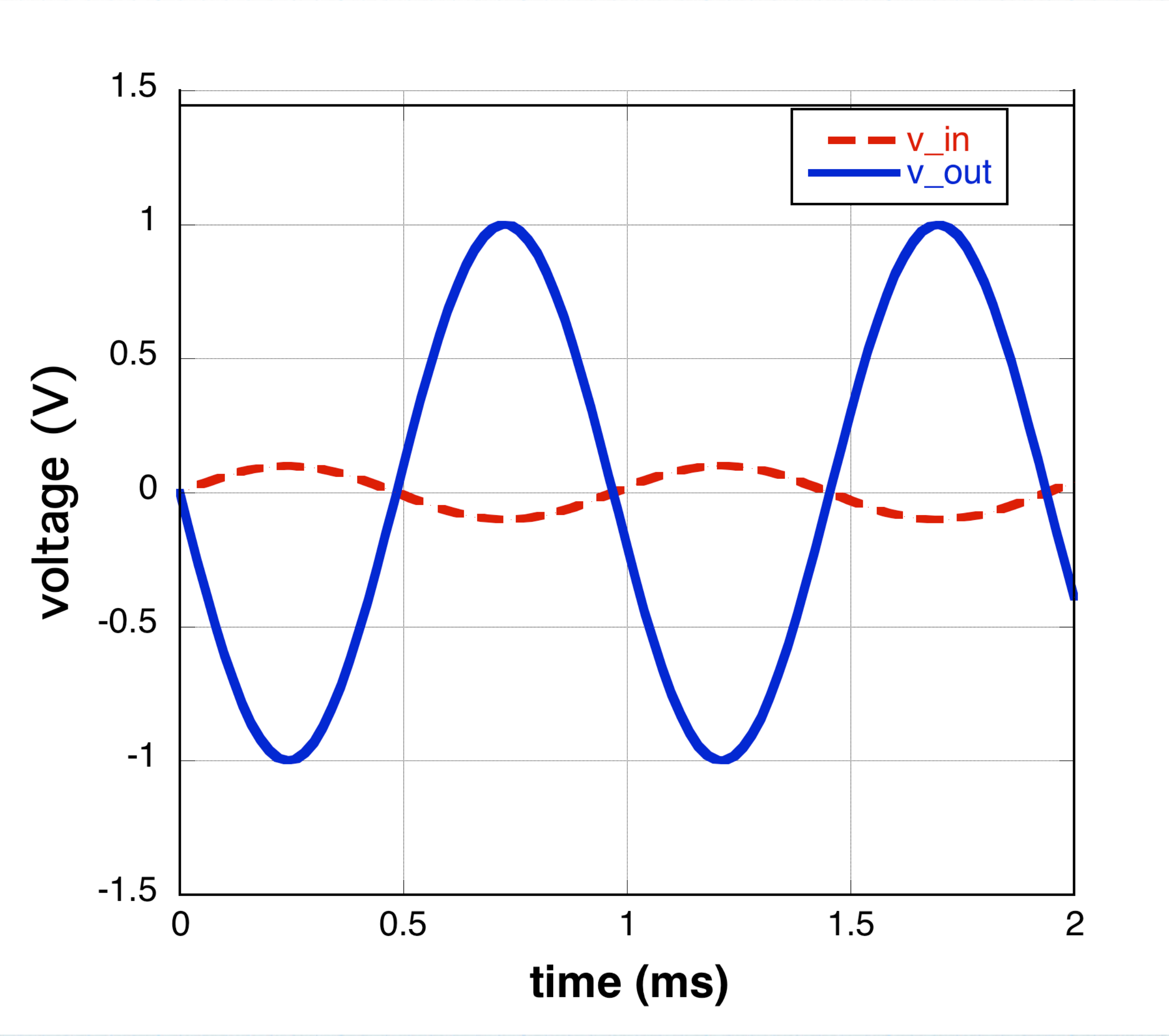
Maybe not that big!!

Generally, it is a good idea to keep the resistors used in op-amp circuits bigger than 1 kΩ.

Non-inverting with gain of 10 (R_2 = 9 kΩ and R_1 = 1kΩ)



Inverting with gain of –10 (R_2 = 10 kΩ and R_1 = 1kΩ)



Summing amp - Digital-to-analog converter (DAC)

$V_{REF} = -1$ V

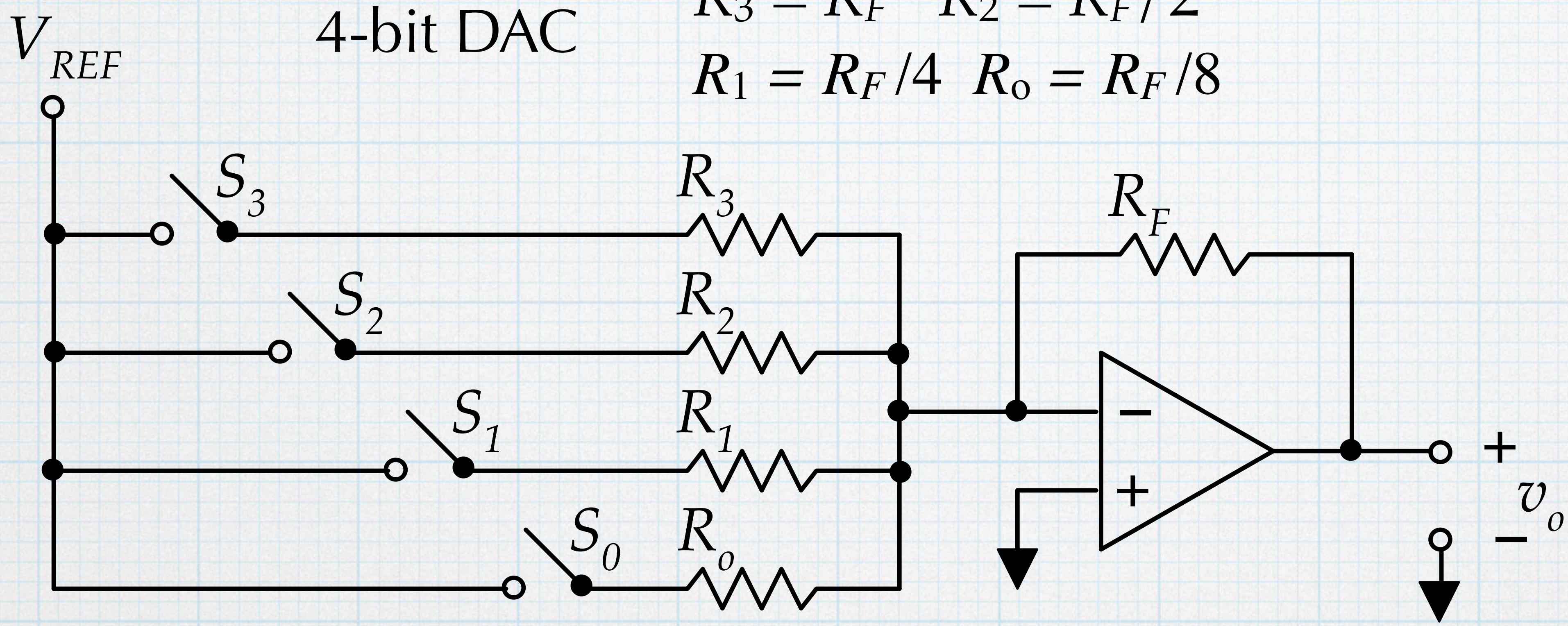
S	v_o (V)
0000	0
0001	0.125
0010	0.25
0011	0.375
0100	0.5
0101	0.625
0110	0.75
0111	0.875
1000	1
1001	1.125
1010	1.25
1011	1.375
1100	1.5
1101	1.625
1110	1.75
1111	2

Data is stored as digital information on your phone or computer. Sometimes that data needs to be converted to analog form in order to use it, i.e. music.

4-bit DAC

$R_3 = R_F \quad R_2 = R_F/2$

$R_1 = R_F/4 \quad R_o = R_F/8$



The digital bits control the operation of the switches – one bit per switch. $S_i = 0$, switch is open, $S_i = 1$ switch is closed.

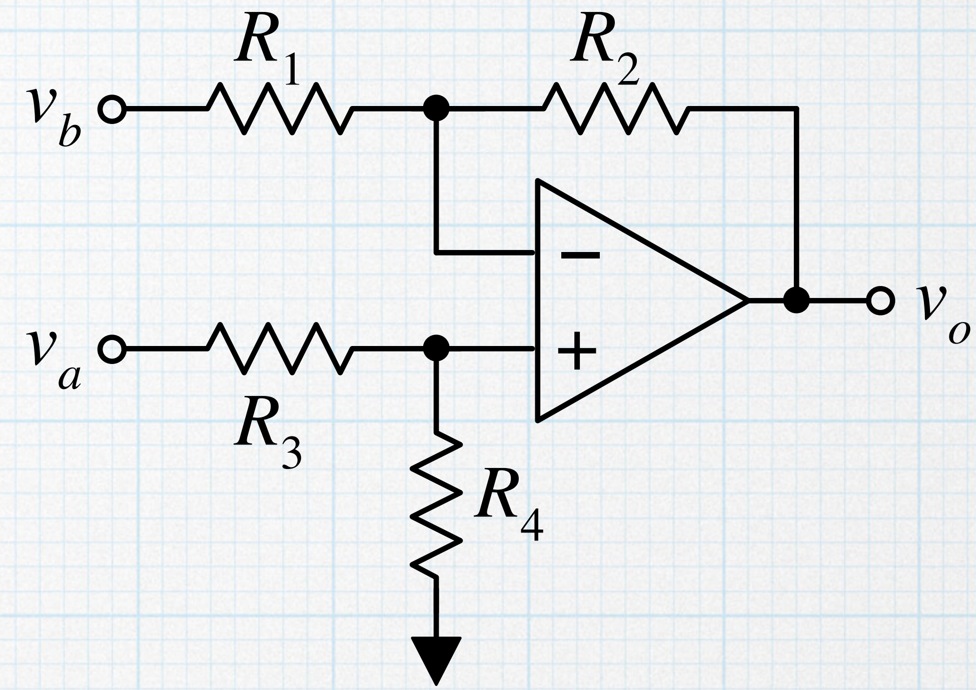
S_o only closed: $v_o = -V_{REF}/8$. S_1 only closed: $v_o = -V_{REF}/4$.

S_2 only closed: $v_o = -V_{REF}/2$. S_3 only closed: $v_o = -V_{REF}$.

Combinations of switched will be summed, and the scaling matches that of a digital number.

Difference amp

Recall the difference amp:



$$v_o = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) v_a - \frac{R_2}{R_1} v_b$$

if $\dfrac{R_2}{R_1} = \dfrac{R_4}{R_3}$

$$v_o = \frac{R_2}{R_1}\left(v_a - v_b\right)$$

Difference only!

The performance of the difference amp depends critically on the matching of the resistors. To see more clearly how the difference amp distinguishes between common and difference signals, we can express v_a and v_b in those terms.

$$v_a = v_{com} + \frac{v_{dif}}{2} \qquad v_b = v_{com} - \frac{v_{dif}}{2} \longrightarrow v_a - v_b = v_{dif}$$

$$v_o = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)\left(v_{com} + \frac{v_{dif}}{2}\right) - \frac{R_2}{R_1}\left(v_{com} - \frac{v_{dif}}{2}\right)$$

$$= \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)v_{dif} + \left[\left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) - \frac{R_2}{R_1}\right]v_{com}$$

$$= G_d \cdot v_{dif} + G_c \cdot v_{com}$$

$$G_d = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)$$

Difference-mode gain.
Goes to R_2/R_1 if $R_2/R_1 = R_4/R_3$.

$$G_c = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) - \frac{R_2}{R_1}$$

Common-mode gain.
Goes to 0 if $R_2/R_1 = R_4/R_3$.

Common-mode rejection ratio

A commonly used measure of how well the difference amp suppresses the common signal is the *common-mode rejection ratio* (CMRR). It is defined as the ratio of the difference-mode gain to the common-mode gain.

$$\text{CMRR} = \left| \frac{G_d}{G_c} \right|$$

Inserting the two gain expressions from the previous page:

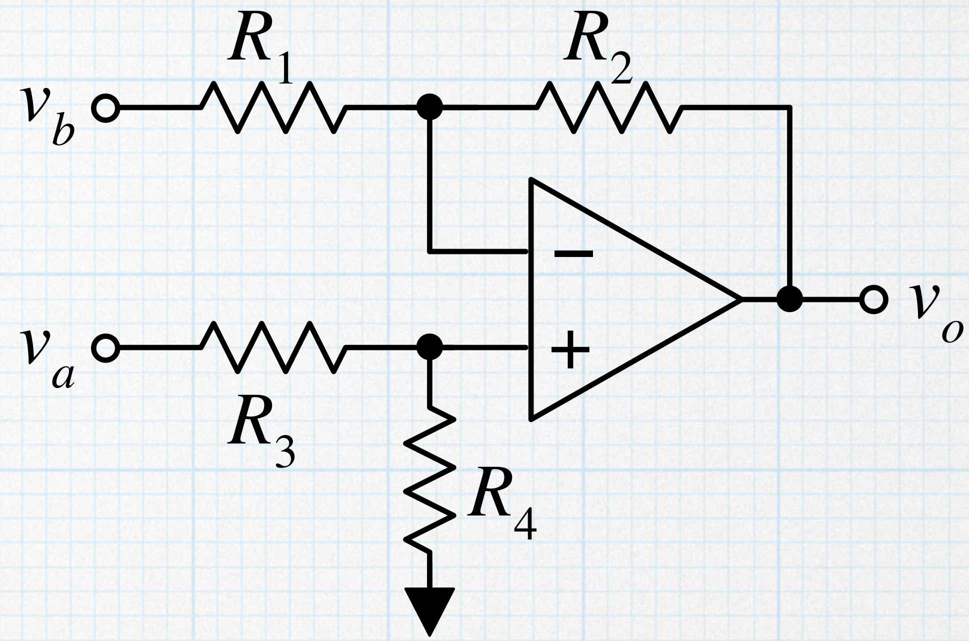
$$\text{CMRR} = \left| \frac{\left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)}{\left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) - \frac{R_2}{R_1}} \right|$$

If all resistor ratios are perfectly matched, $\text{CMRR} = \frac{\frac{R_2}{R_1}}{0} \rightarrow \infty$. So we expect CMRR to be large.

Suppose matching is not perfect. For example, if $R_2 / R_1 = 10$ and $R_4 / R_3 = 9.9$, then CMRR = 1089 — a big number, but definitely not infinity!

Often, CMRR is expressed using decibels. For example CMRR = 1089 = 60.7 dB.

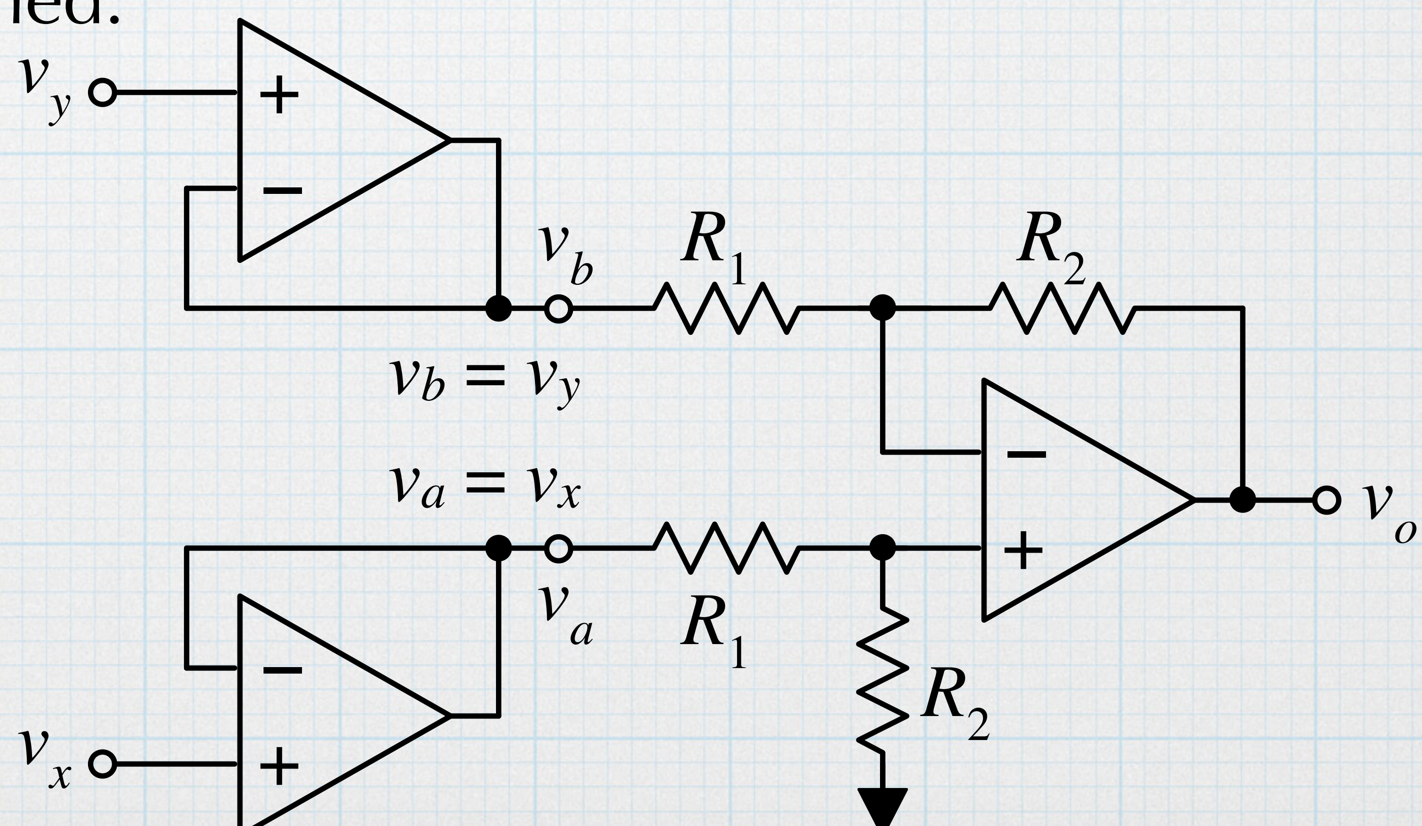
Instrumentation amplifier



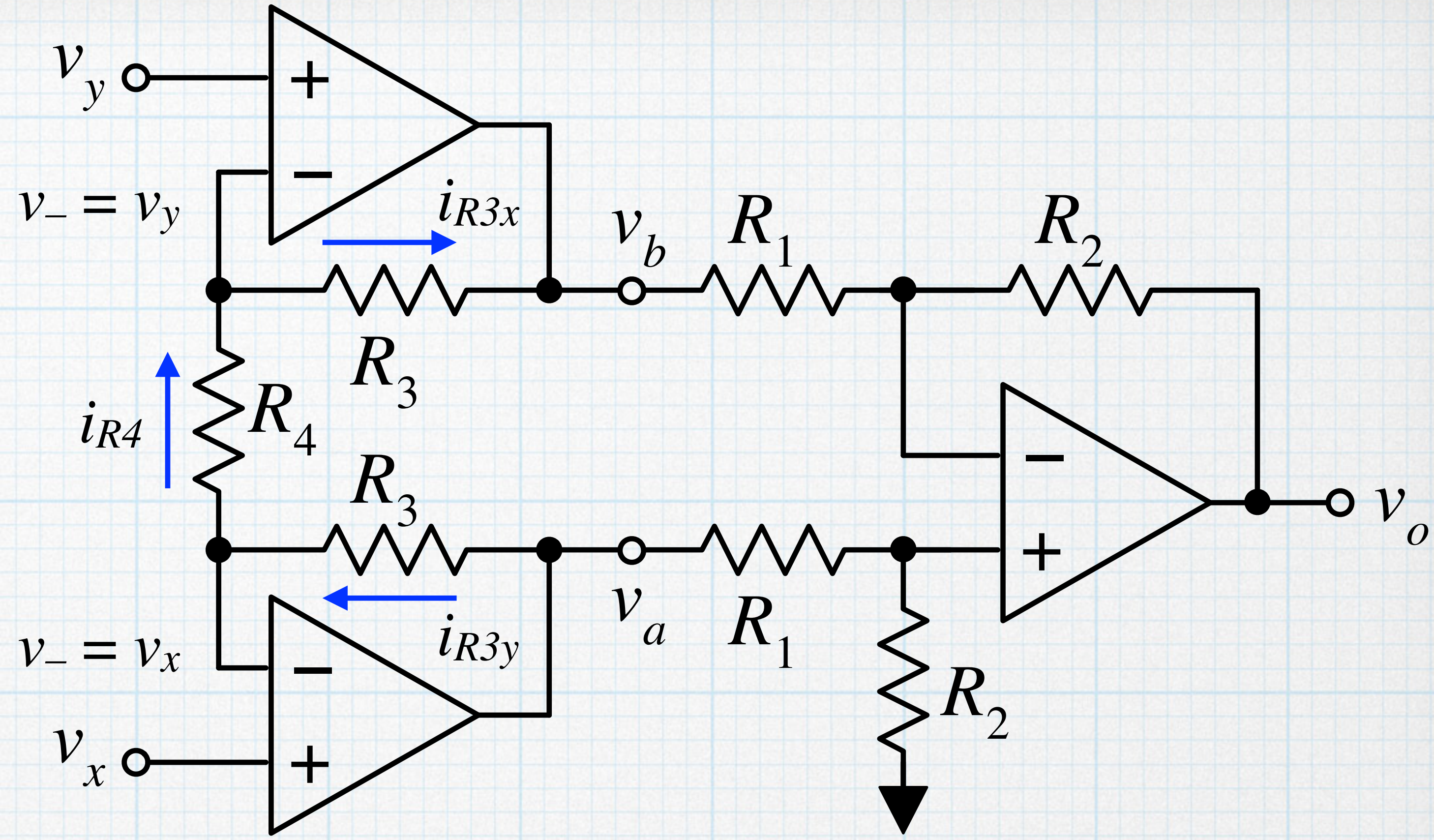
Recall the difference amp: If the ratios are matched ($R_4/R_3 = R_2/R_1$) then the circuit becomes a serviceable difference amp with $v_o = G_d(v_a - v_b)$ and high CMRR.

But it could be better. First, the input resistances seen by v_a and v_b depend on the values of R_1 and R_3, which might be smallish. (See earlier slide on input reistance. Secondly, it might be nice to be able adjust the gain using a potentiometer, but that is difficult because of the requirement that ratios be matched.

First improvement: add unity-gain buffers to each input. The input resistances go to infinity (ideally) without changing the gain or the CMRR.



A second modification leads to the instrumentation amplifier. Add two matched resistors R_3 and a single resistor R_4 to turn the two input amps in "pseudo-non-inverting" amps.



We note some things based on the "op amp rules".

$$v_{R4} = v_x - v_y \quad \rightarrow \quad i_{R4} = \frac{v_x - v_y}{R_4}$$

$$i_{R3x} = i_{R3y} = i_{R4}$$

Now, put it all together.
Use KVL, starting at node a.

$$v_a - i_{R3x}R_3 - i_{R4}R_4 - i_{R3y}R_3 = v_b$$

$$v_a - v_b = i_{R4}\left(2R_3 + R_4\right)$$

$$v_a - v_b = \left(\frac{2R_3}{R_4} + 1\right)\left(v_x - v_y\right)$$

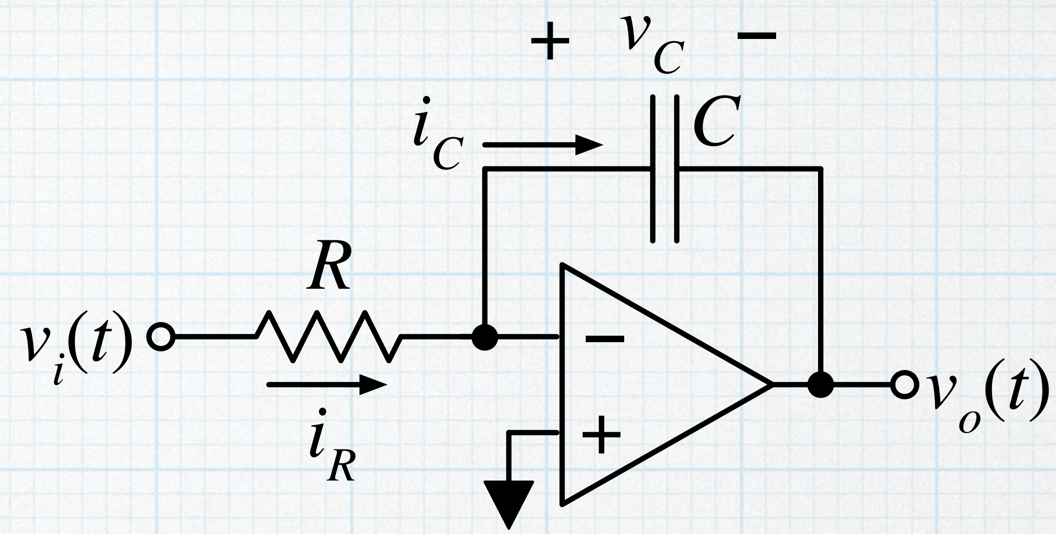
$$v_o = \left(\frac{R_2}{R_1}\right)\left(v_a - v_b\right) \qquad \text{(Still true!)}$$

$$v_o = \left(\frac{R_2}{R_1}\right)\left(\frac{2R_3}{R_4} + 1\right)\left(v_x - v_y\right)$$

Still a difference amp. If R_4 is a potentiometer, the gain is variable!

Integrating amplifier

Consider an inverting amp with a capacitor as the feedback element.



$$i_R = i_C$$

$$\frac{v_i(t)}{R} = C\frac{dv_C(t)}{dt}$$

$$v_C = -v_o$$

$$\frac{v_i(t)}{R} = -C\frac{dv_o(t)}{dt}$$

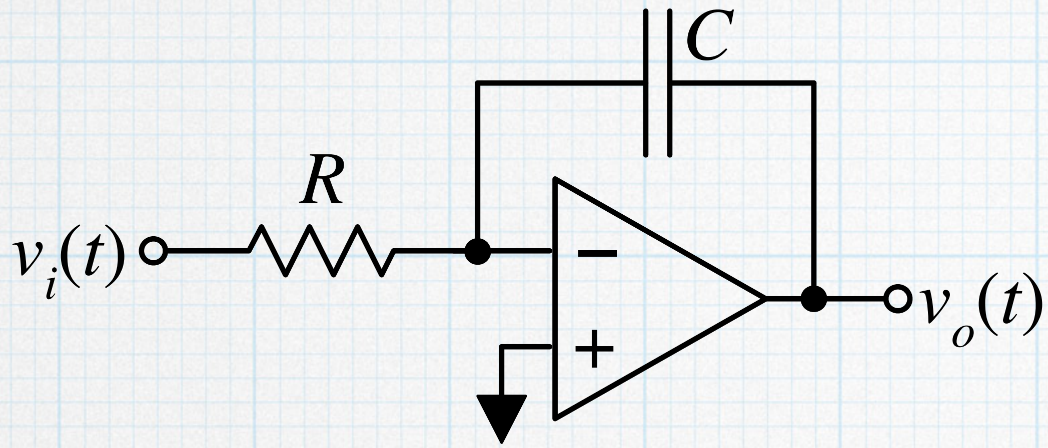
$$dv_o = -\frac{1}{RC}v_i(t)\,dt$$

$$\int_{v_o(0)}^{v_o(t)} dv_o' = -\frac{1}{RC}\int_0^t v_i(t')\,dt'$$

$$v_o(t) - v_o(0) = -\frac{1}{RC}\int_0^t v_i(t')\,dt'$$

$$\boxed{v_o(t) = -\frac{1}{RC}\int_0^t v_i(t')\,dt' + v_o(0)}$$

The output as a function of time is the integral over time of the input.

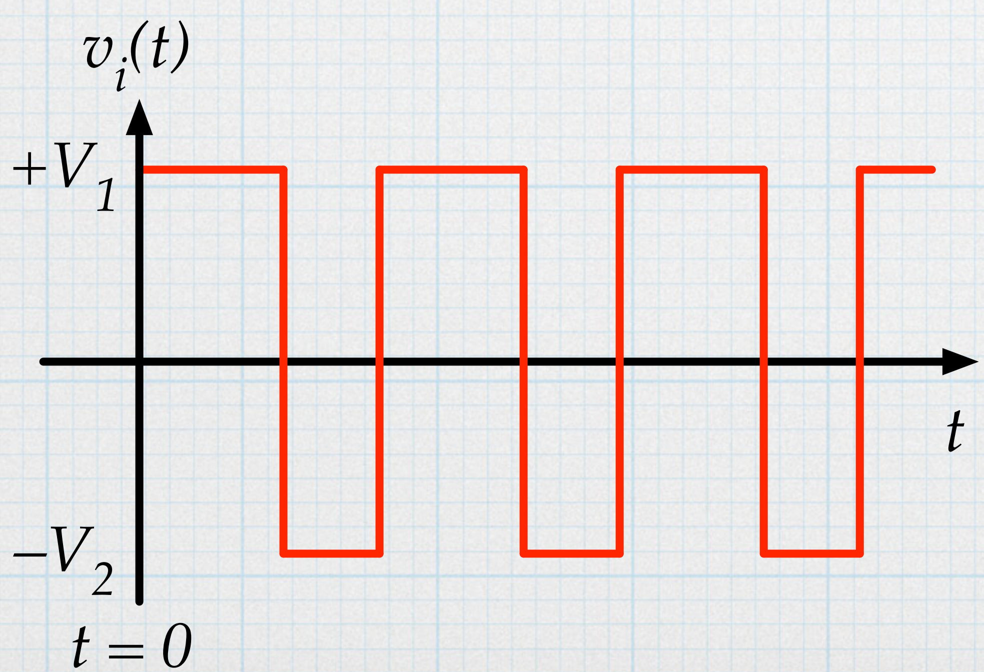


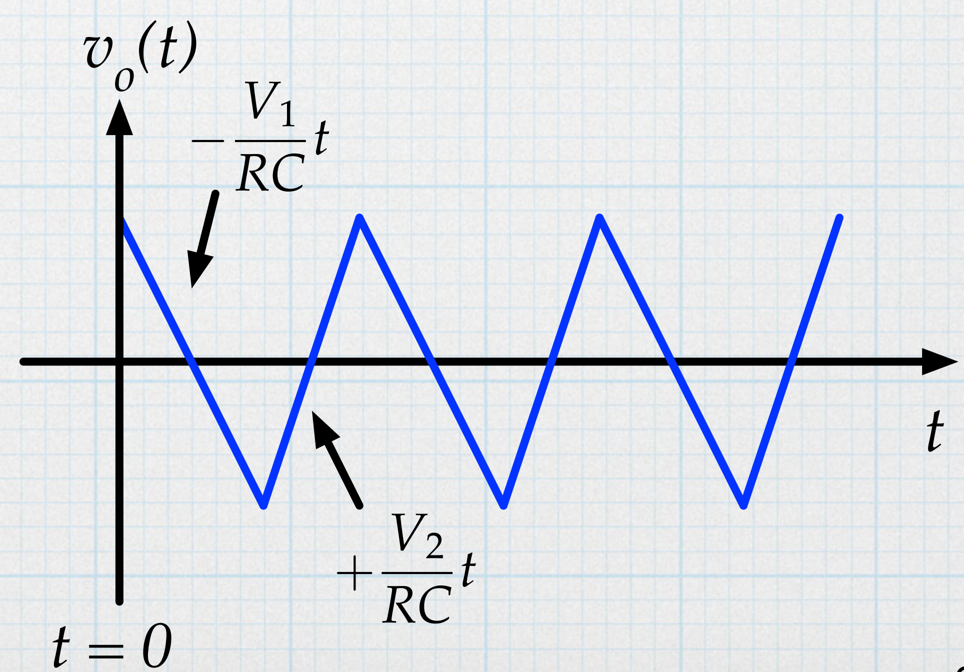
$$v_o(t) = -\frac{1}{RC}\int_0^t v_i(t')\,dt' + v_o(0)$$

If the input is a constant voltage, $v_i(t) = V_1$, then $v_o(t) = -\frac{V_1}{RC}t + v_o(0)$

The output starts at whatever value it has t = 0, and then *ramps* in time with slope $-V_1/RC$. If V_1 is positive, the output ramps down in time, and if V_1 is negative, then the output ramps upward.

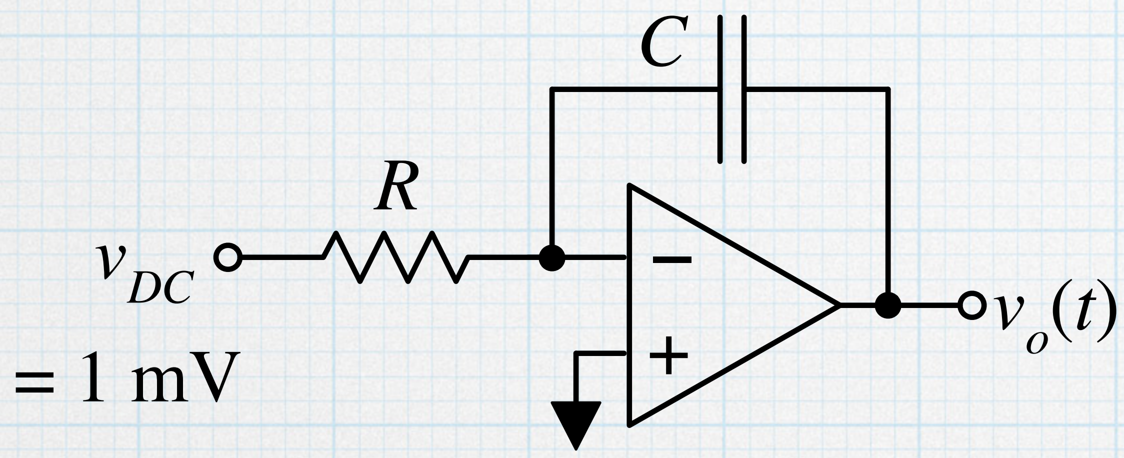
If the input switches back and forth between two constant values (square wave), then the output ramps up and down correspondingly (sawtooth).

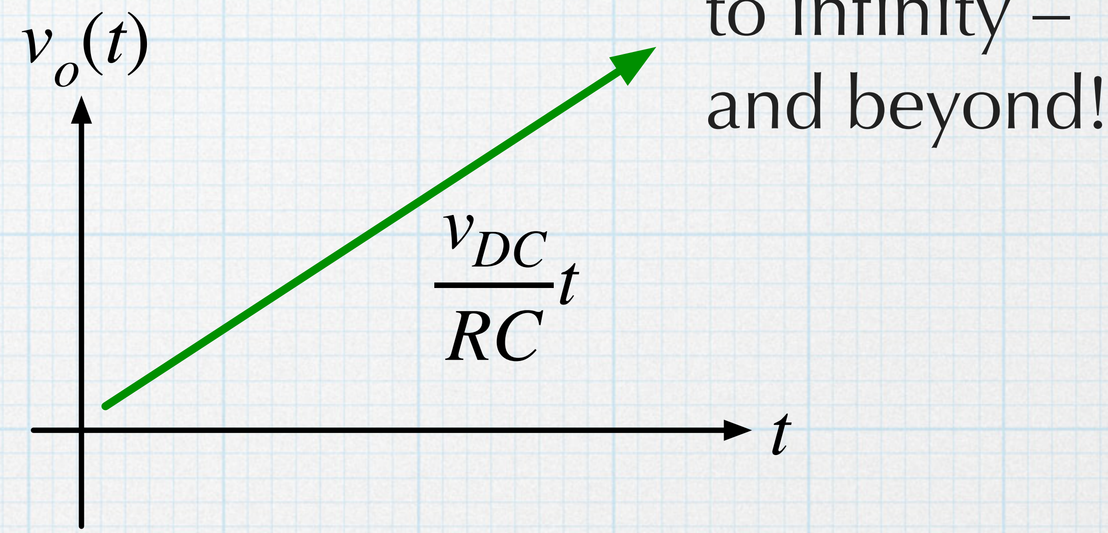




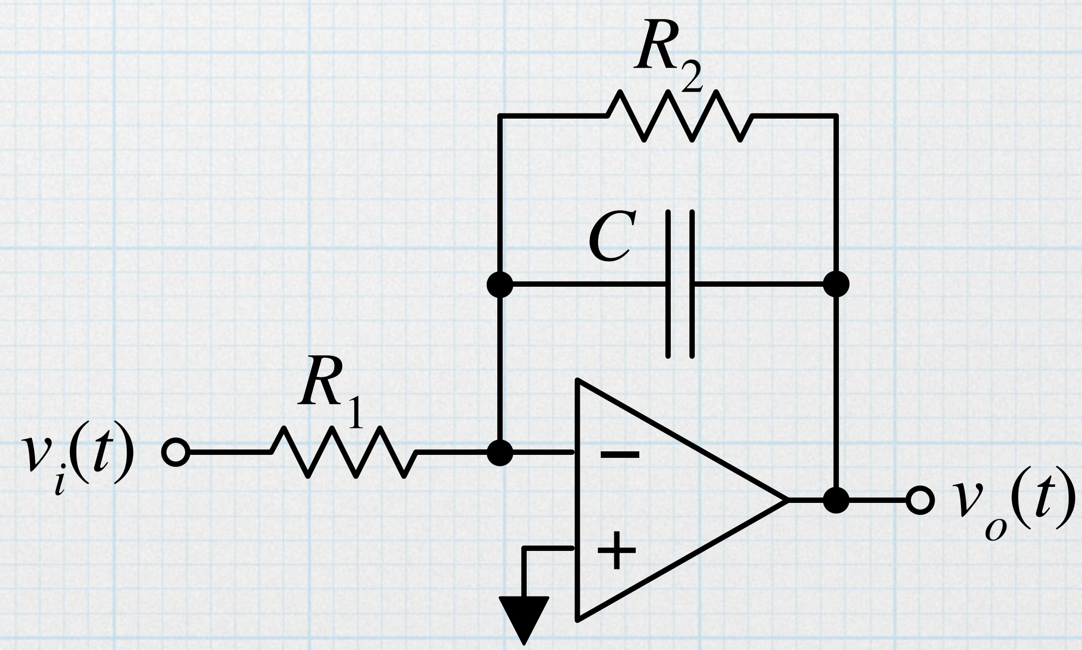
A practical concern with integrating amps:

If there is a small, but constant, DC voltage at the input (and we will see later that most op-amps come with built-in DC error voltages at the inputs), then that voltage will be integrated forever and the output will go to infinity. (In reality, it will saturate at the power supply limit.)



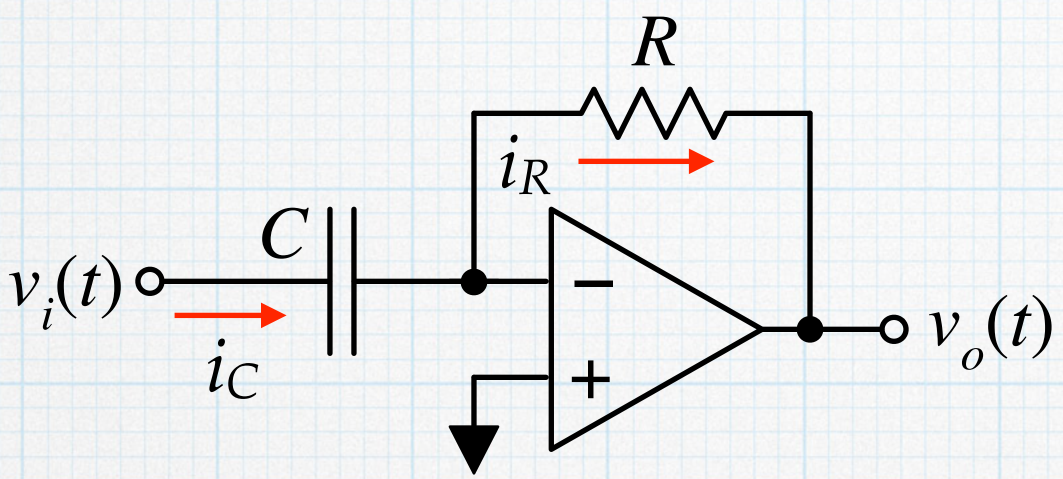


This is a problem because at DC, the capacitor is an open circuit and the amplifier has essentially infinite DC gain. To make it better, put a resistor in parallel with the cap, $R_2 >> R_1$. This limits the DC gain to $-R_2/R_1$.



Differentiating amplifier

Op amps can also differentiate. Switch resistor and capacitor.



$i_C = i_R$ $\qquad\qquad$ $v_C = v_i$

$$C\frac{dv_C}{dt} = \frac{-v_o}{R}$$

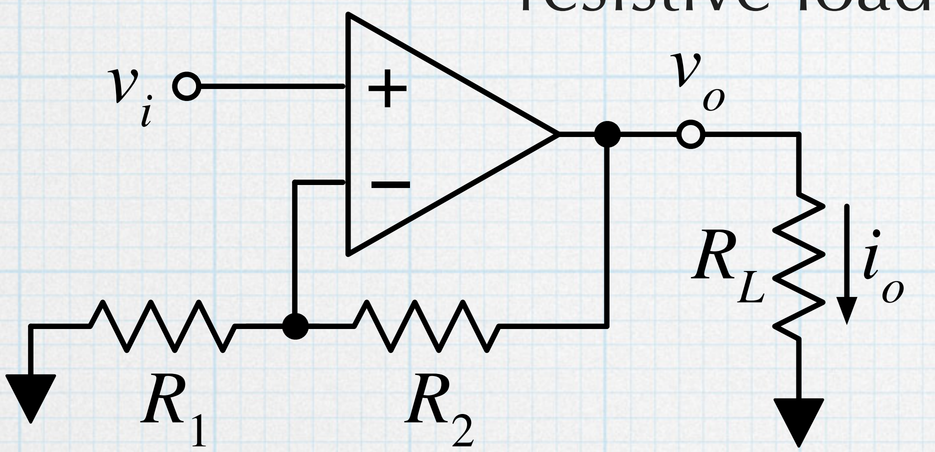
$$v_o(t) = RC\frac{dv_i}{dt}$$

However, differentiating amps are not used much. If there is noise at the input (characterized by random, fast variations), the differentiator tends to make it worse. The integrator, on the other hand, tends to average out the effects of noise.

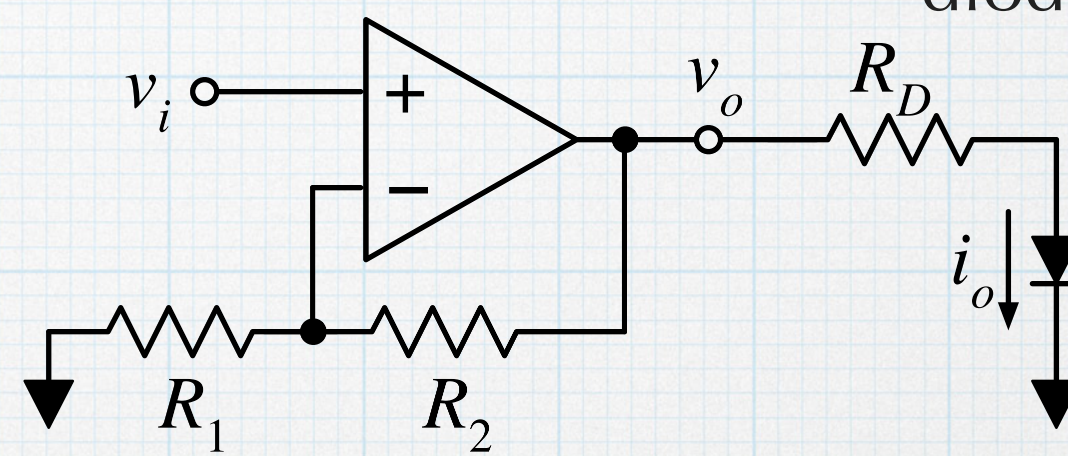
Loading and cascading

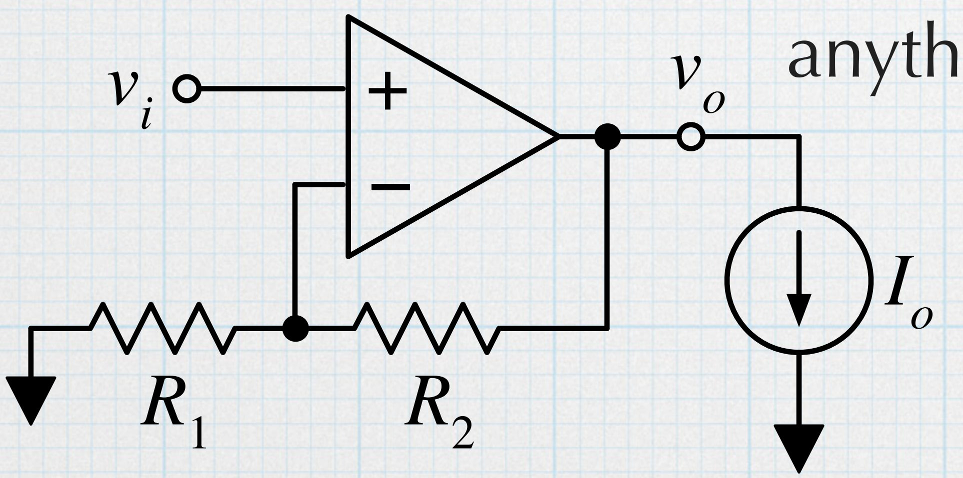
An ideal amp has an (ideal) dependent voltage source connected directly to the output with no output resistance. So the ideal amp will be able to source any amount of current, and it will not matter what load is attached.

resistive loads



diodes





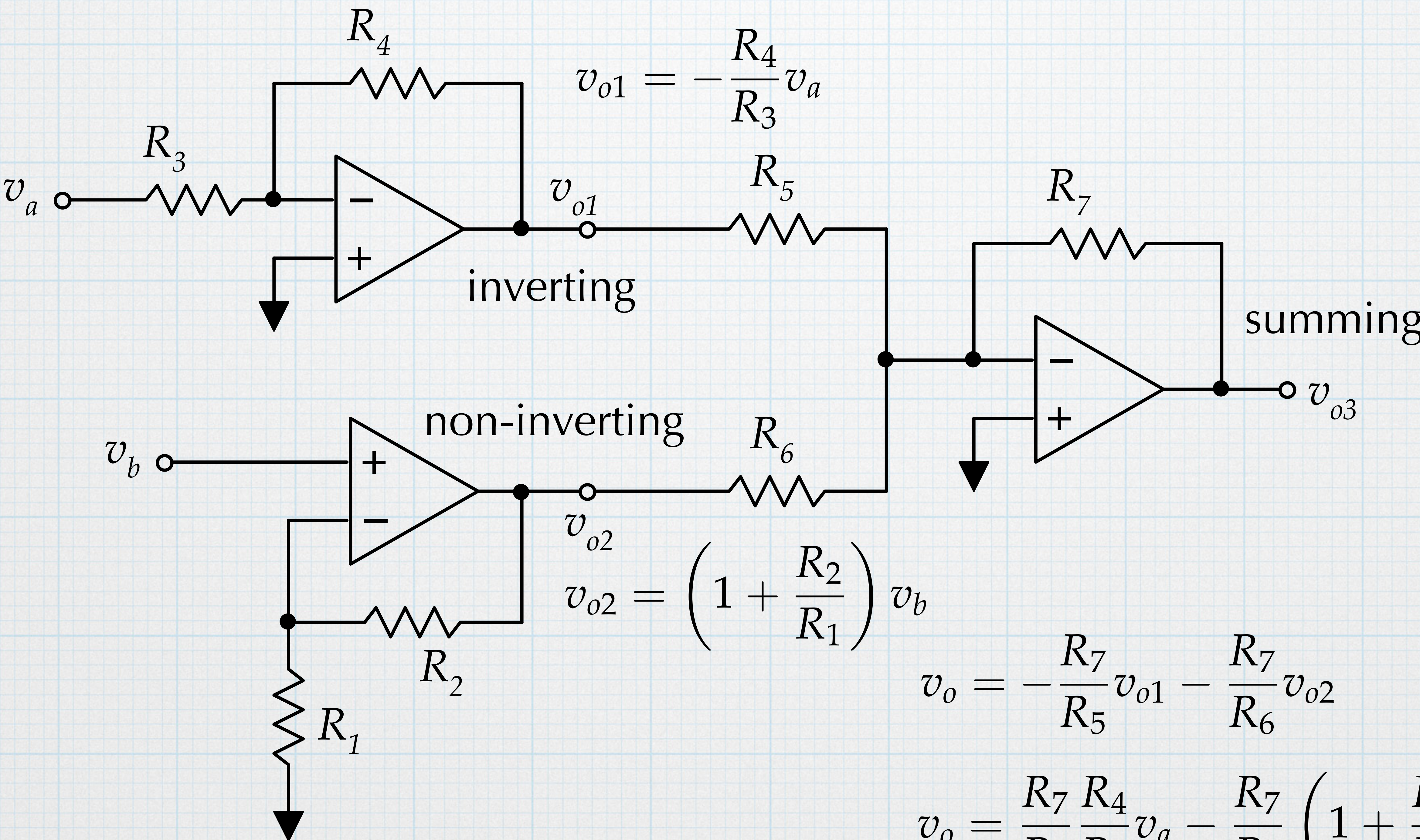
anything!

In all cases: $v_o = \left(1 + \frac{R_2}{R_1}\right) v_i$

We will learn soon enough that this is a gross over-simplification. There is not unlimited current available, and we will have to be careful about how our op amps are loaded.

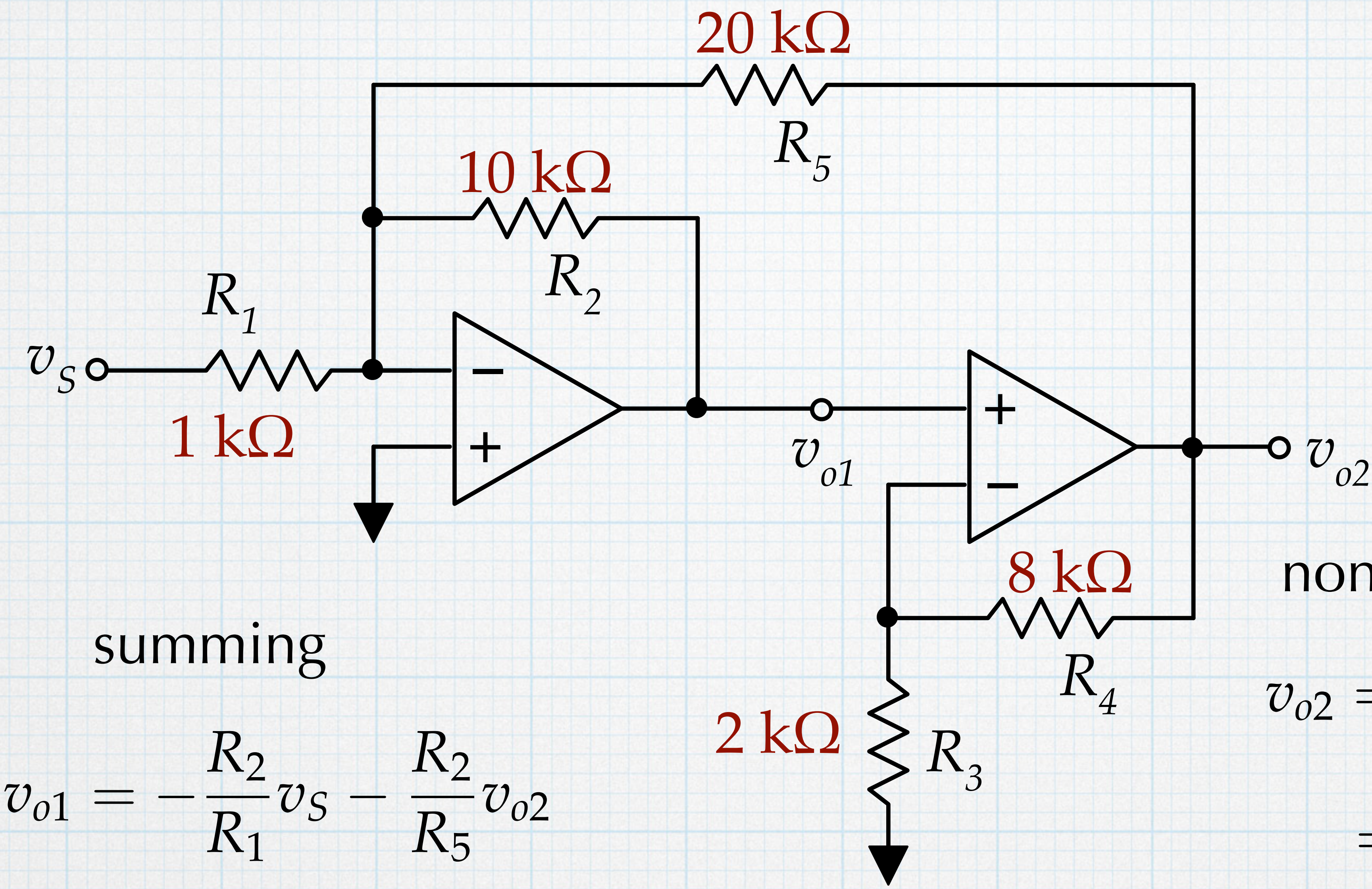
Cascading amps

Zero output resistance means that we can create the various types of circuits as building blocks for more complicated circuits.



$$v_o = -\frac{R_7}{R_5} v_{o1} - \frac{R_7}{R_6} v_{o2}$$

$$v_o = \frac{R_7}{R_5}\frac{R_4}{R_3} v_a - \frac{R_7}{R_6}\left(1 + \frac{R_2}{R_1}\right) v_b$$



non-inverting

$$v_{o2} = \left(1 + \frac{R_4}{R_3}\right) v_{o1}$$

$$= 5 v_{o1}$$

summing

$$v_{o1} = -\frac{R_2}{R_1} v_S - \frac{R_2}{R_5} v_{o2}$$

$$= -10 v_S - 0.5 v_{o2}$$

want v_{o2} / v_S.

$$\frac{v_{o2}}{5} = -10 v_S - 0.5 v_{o2}$$

$$0.7 v_{o2} = -10 v_S$$

$$\frac{v_{o2}}{v_S} = -14.29$$

Feedback loop around feedback loops !!