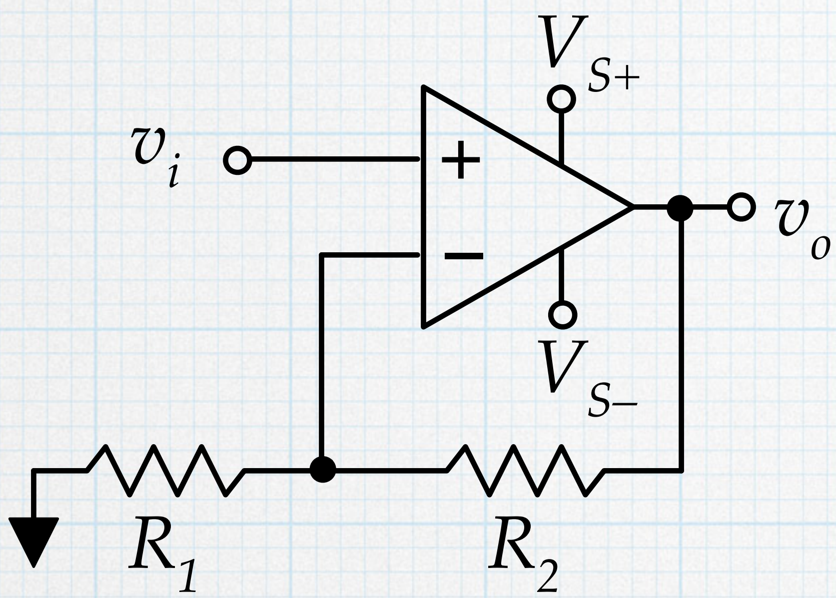
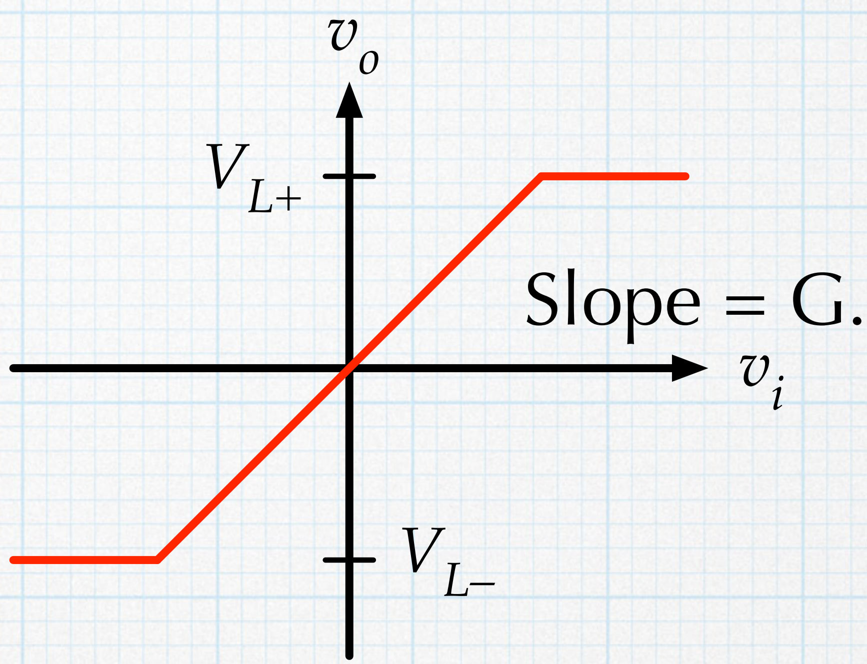
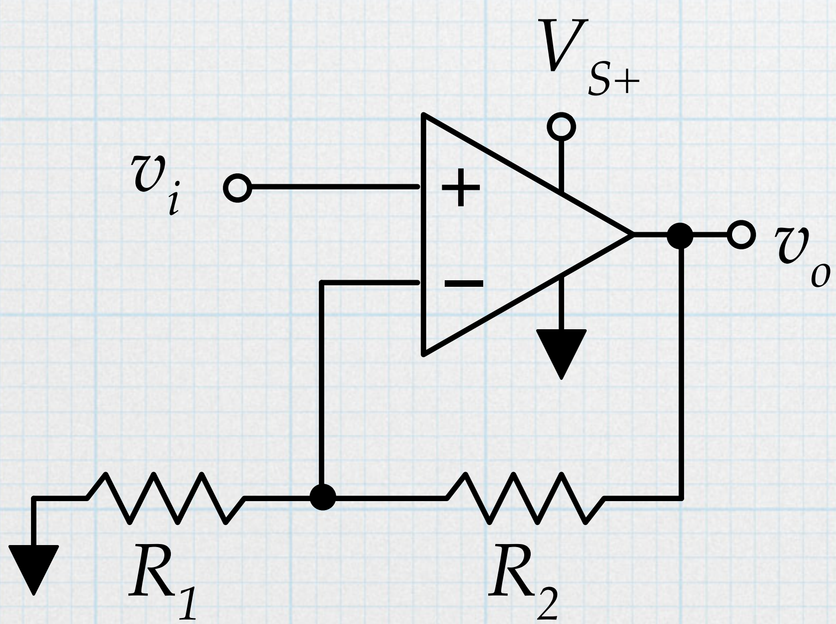
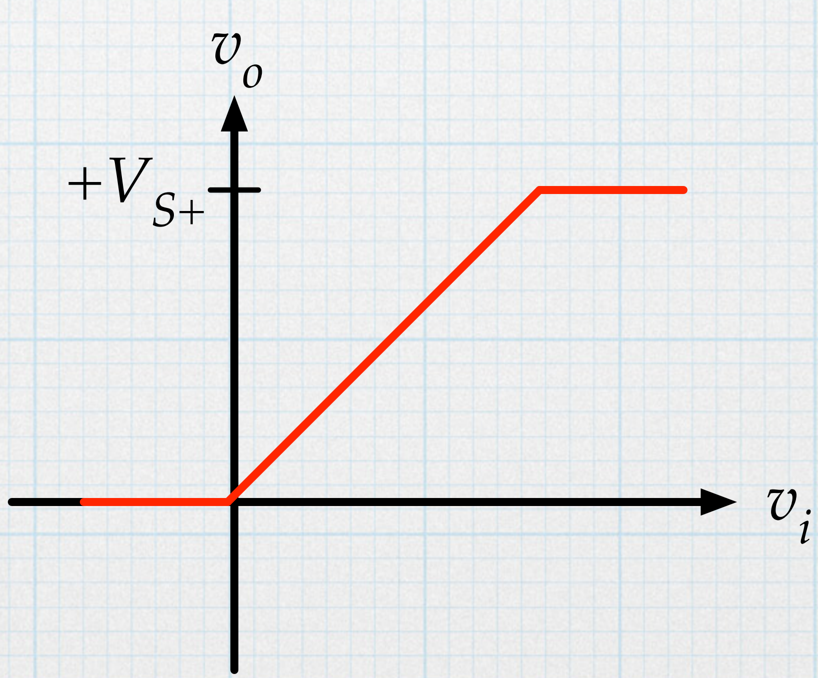
Comparators

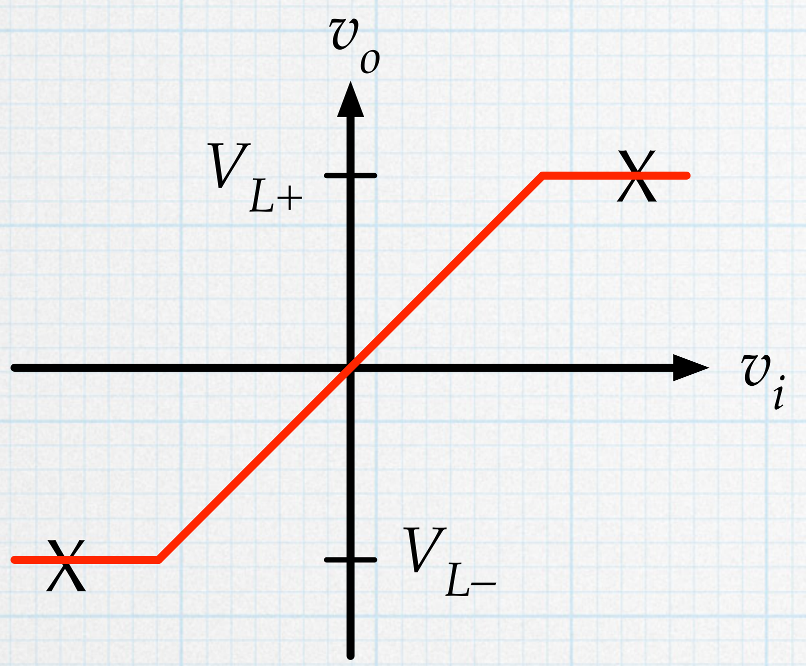




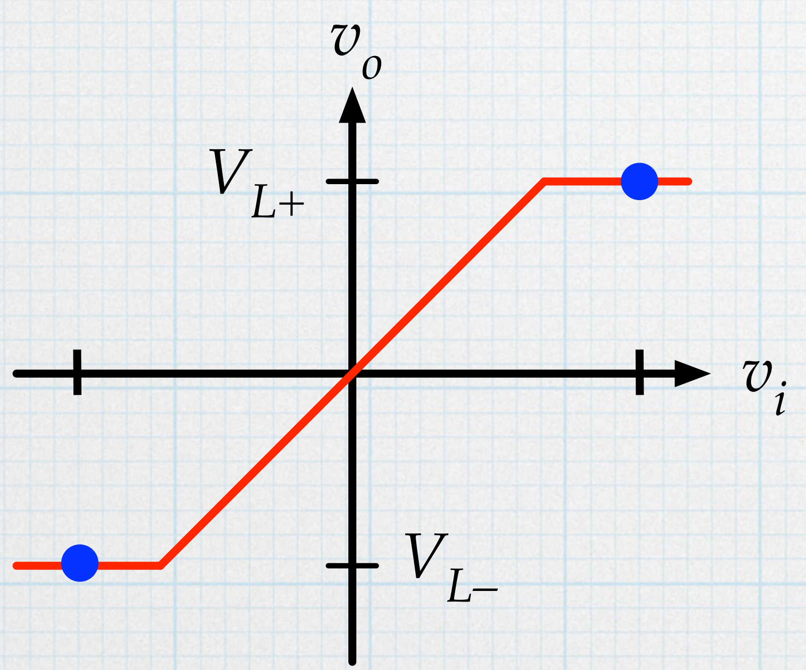
Non-inverting amp. $G = 1 + R_2/R_1$. If the op amp has rail-to-rail outputs, then $V_{L+} = V_{S+}$ and $V_{L-} = V_{S-}$.







For amplifier applications, we try to stay within the linear gain region and avoid operating in the saturated areas. This makes a good amplifier.



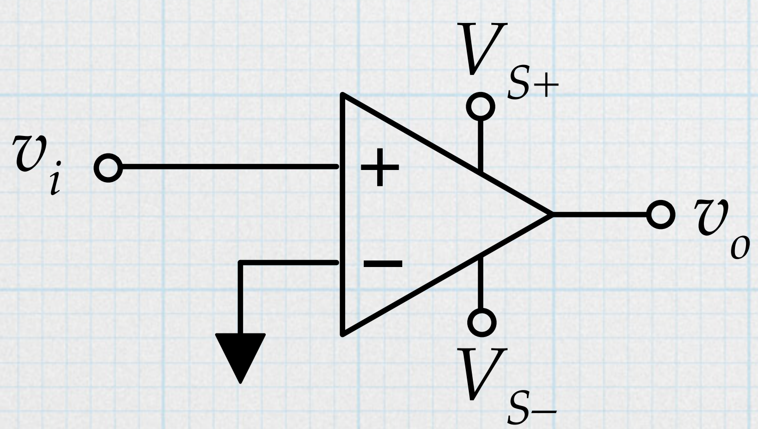
But the saturation regions are not totally useless. For example, if the output is saturated at V_{L+}, then we know that the input must be large(ish) and positive. Or, if the output is saturated at V_{L-}, then we know that the input is large(ish) and negative.

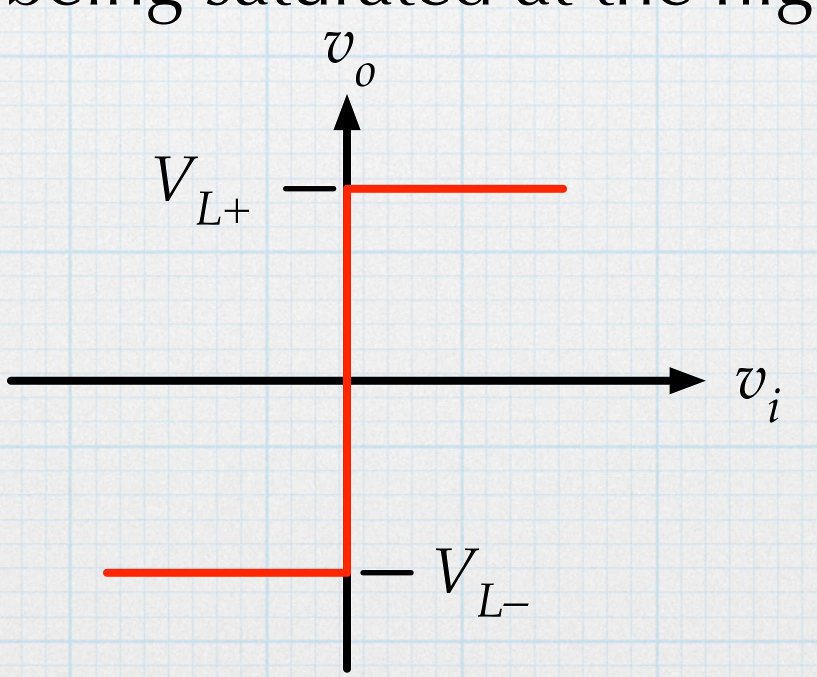
Saturated outputs give some comparative information – the input is either positive or negative.

Using the saturated levels gives us a "yes/no" type of circuit. Is the input voltage high? Yes or no? Is the input voltage low? Yes or no?

The weakness of using a regular feedback amplifier circuit in this way is the gain region is indeterminate. If the input voltage is such that amplifier is working in its linear region, then the answer to our simple question is "Dunno."

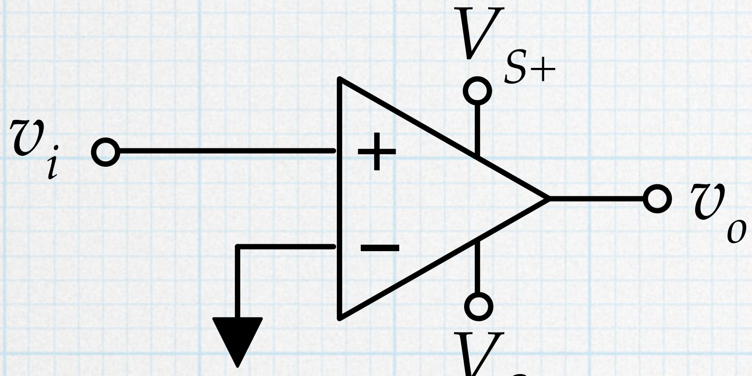
To make the circuit be more "decisive", we should sharpen the gain region transition. This suggests a very simple alternative – remove the feedback loop and use the op amp open loop. With its very high open-loop gain, there will be an extremely sharp transition from being saturated at the low level to being saturated at the high level.

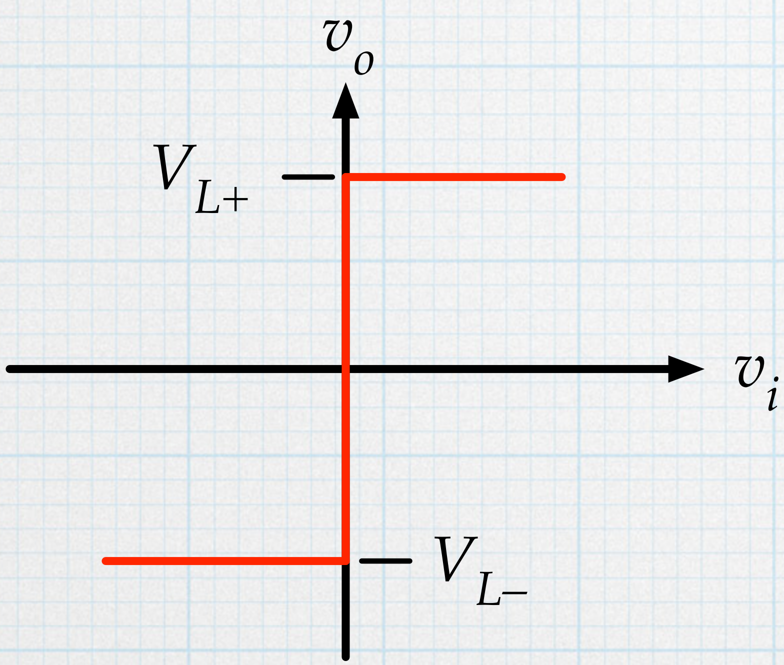




We can call this a comparator.

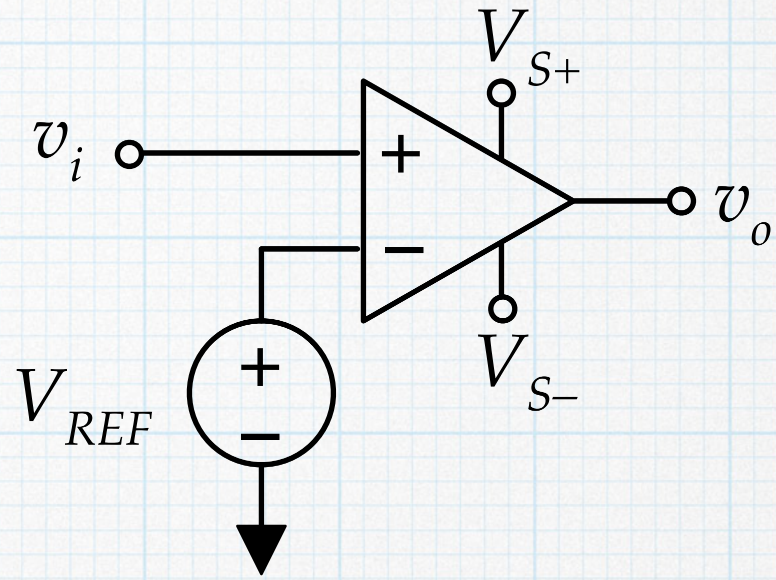
Non-inverting comparator

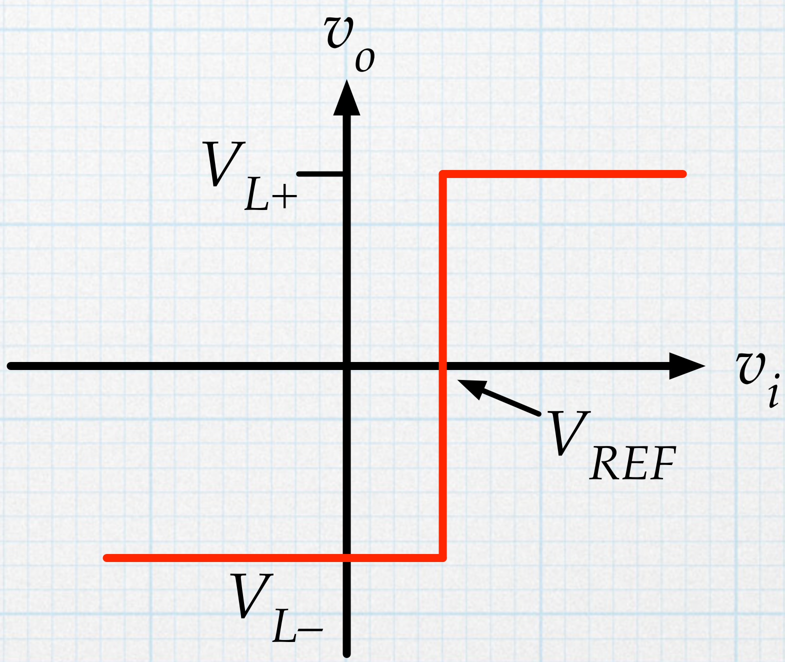




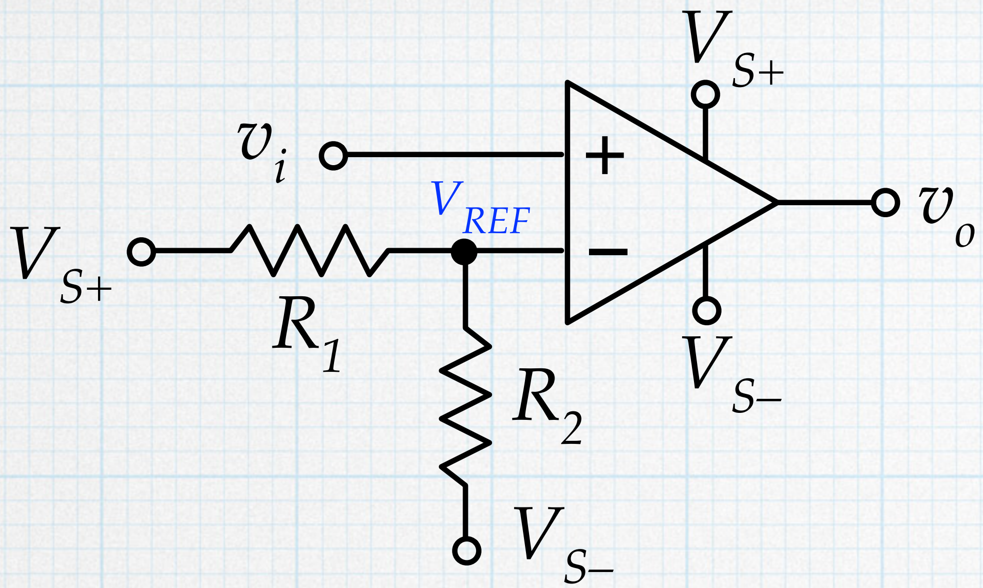
If $v_i > 0$, $v_o = V_{L+}$
if $v_i < 0$, $v_o = V_{L-}$.

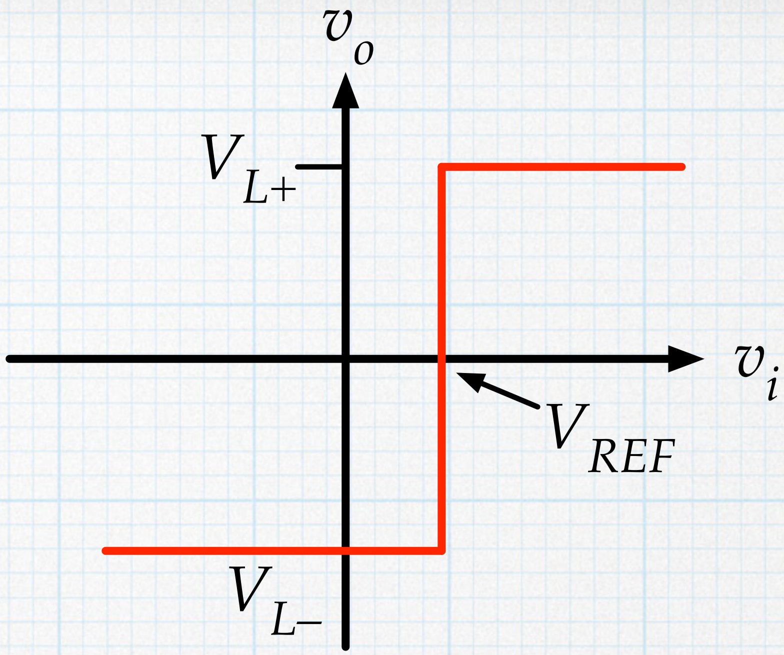
It's that simple.





Shift the comparison point:
If $v_i > V_{REF}$, $v_o = V_{L+}$
if $v_i < V_{REF}$, $v_o = V_{L-}$.

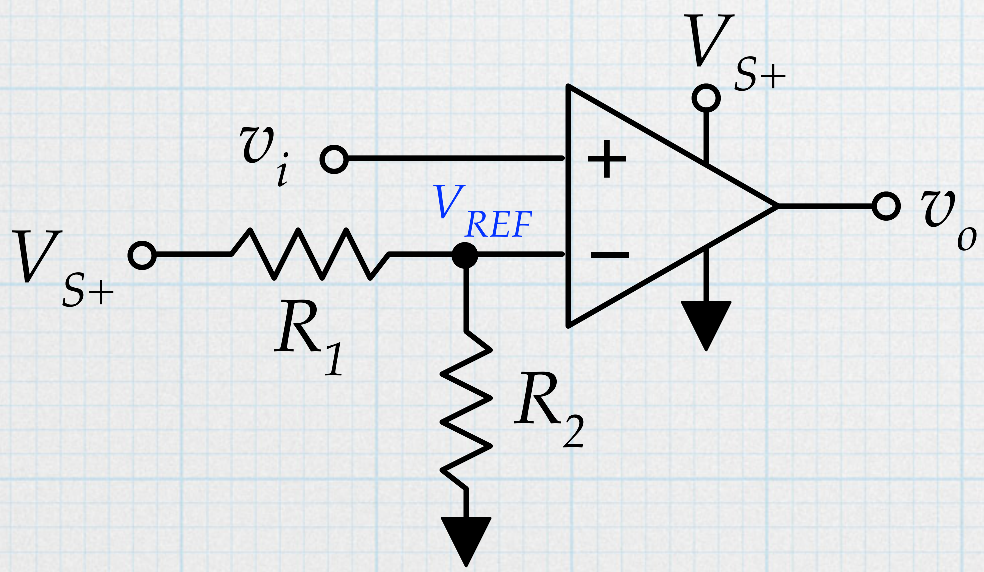


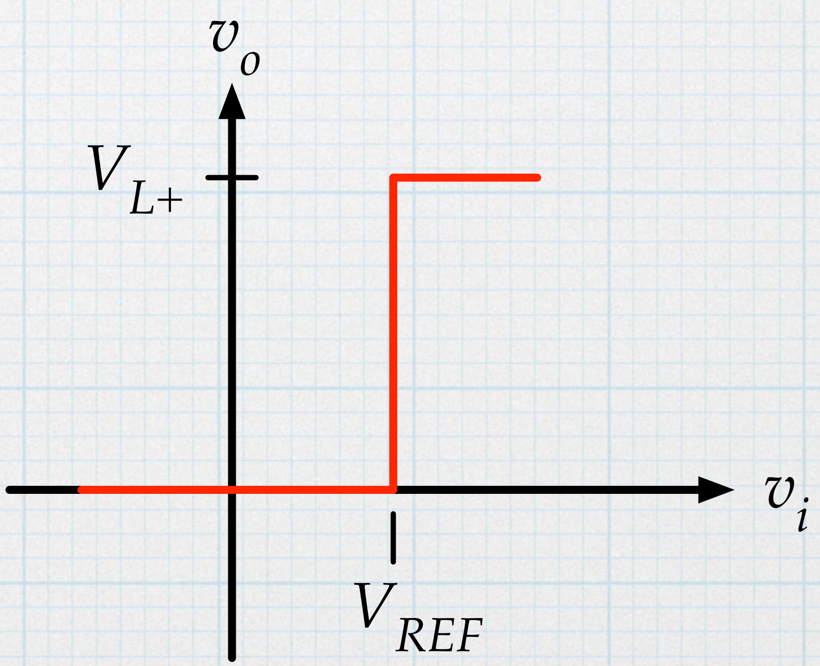


Use a voltage divider. (Perhaps a potentiometer.)

$$V_{REF} = V_{S-} + \frac{R_2}{R_1 + R_2} \left(V_{S+} - V_{S-} \right)$$

Single-supply version.

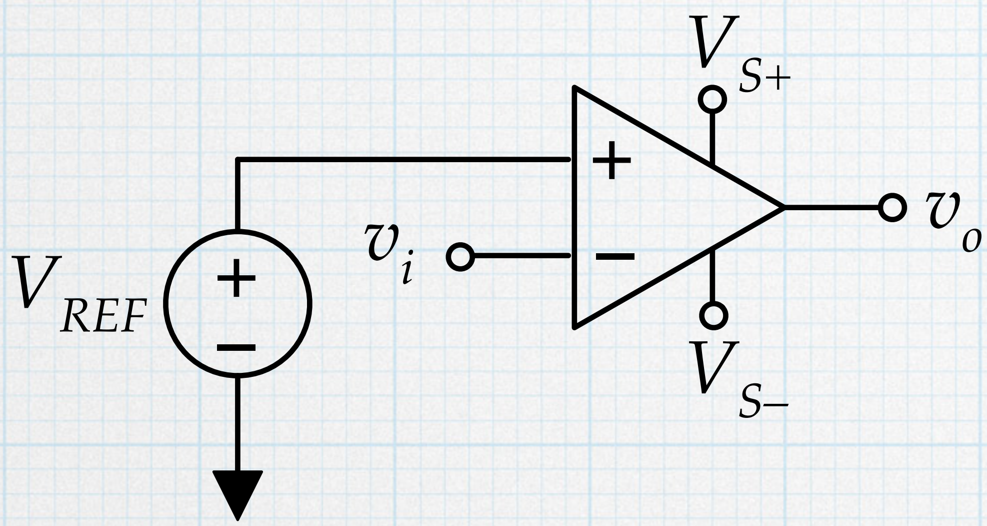


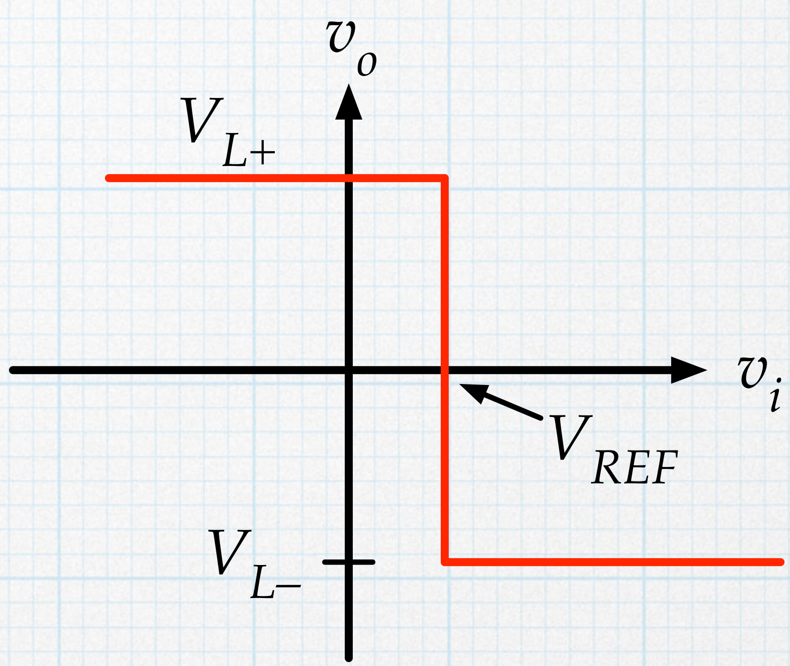


$$V_{REF} = \frac{R_2}{R_1 + R_2} V_{S+}$$

Inverting comparator

Switch the inputs to have it work in the opposite fashion.





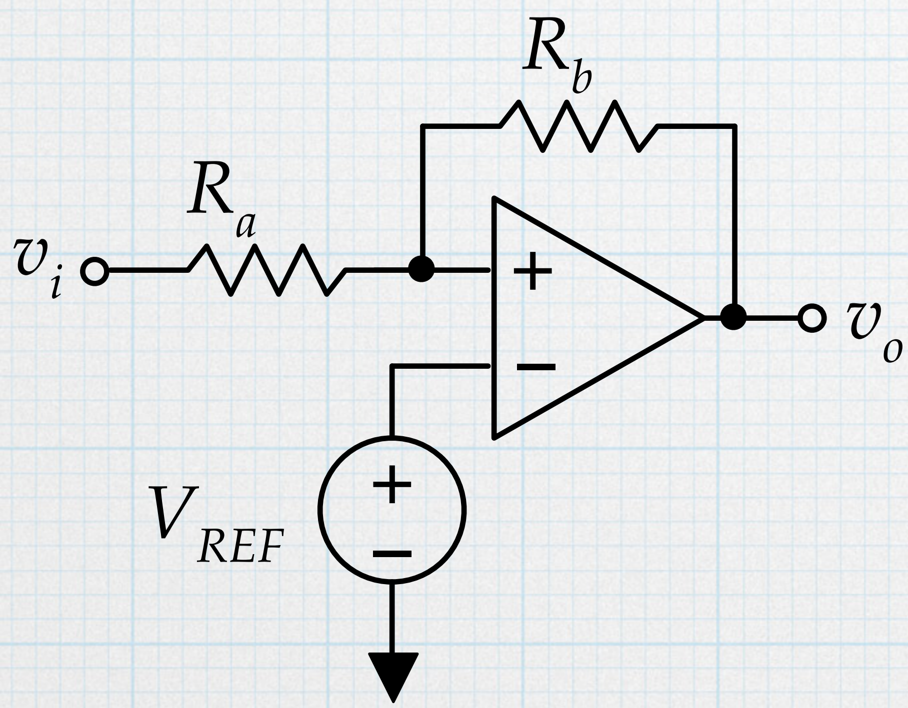
If $v_i > V_{REF}$, $v_o = V_{S-}$

if $v_i < V_{REF}$, $v_o = V_{S+}$.

Use positive feedback to improve performance

Simple open-loop comparators have problem with "chatter". If input signal is noisy and it is close to V_{REF}, the output can bounce back and forth. This is not desirable. We can limit this problem by using positive feedback to introduce *hysteresis*.

Positive feedback also make the output switch even faster from one level to the other.



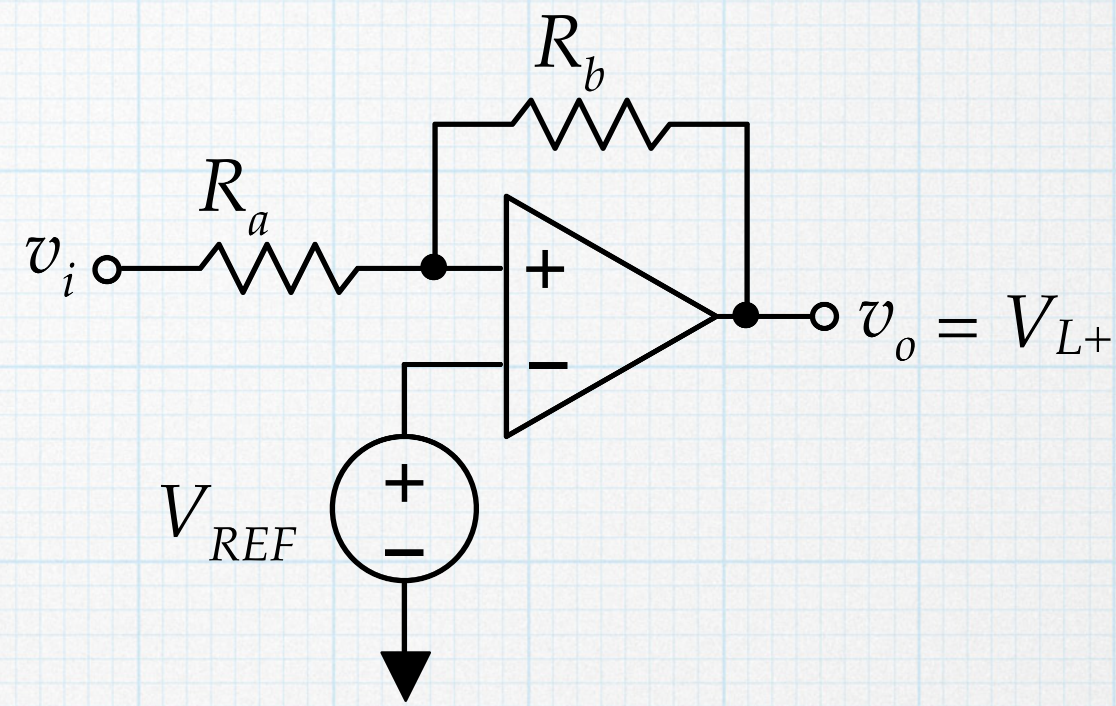
Looks normal enough. But wait, the feedback loop is "backwards".

This is a positive feedback loop. In that case $v_+ \neq v_-$.

If v_i changes, it causes v_+ to change. When v_+ crosses v_- ($=V_{REF}$), output will change.

Extremely non-linear. Have to analyze it piecemeal.

Start by assuming that v_i is positive enough to have $v_+ > V_{REF}$.
Then $v_o = V_{L+}$.



Write a node equation at the non-inverting input.

$$\frac{v_i - v_+}{R_a} = \frac{v_+ - V_{L+}}{R_b}$$

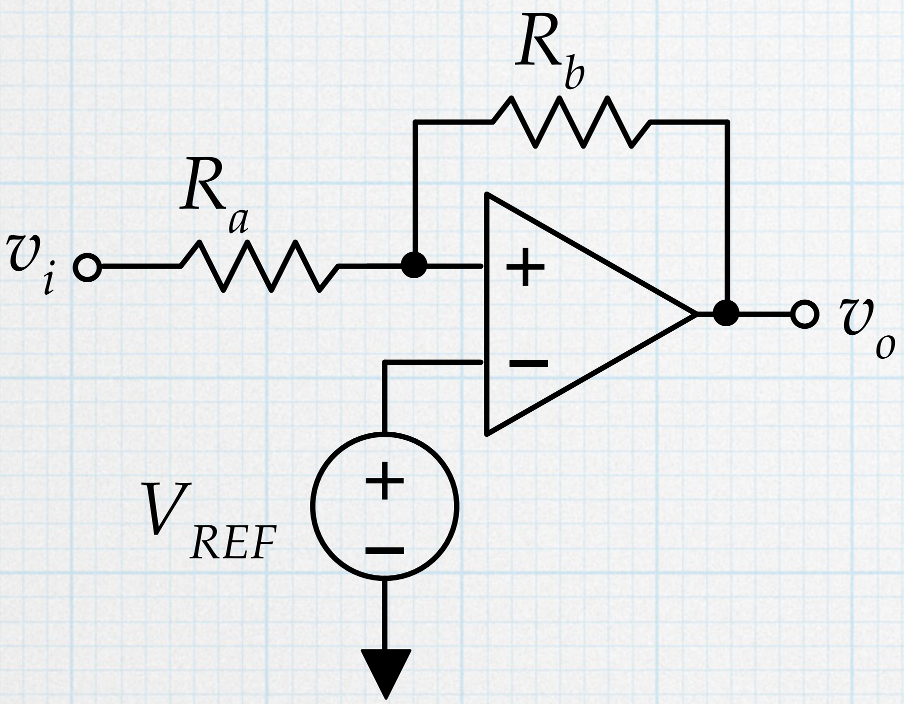
$$v_+ = \frac{v_i + \left(\frac{R_a}{R_b}\right) V_{L+}}{1 + \frac{R_a}{R_b}}$$

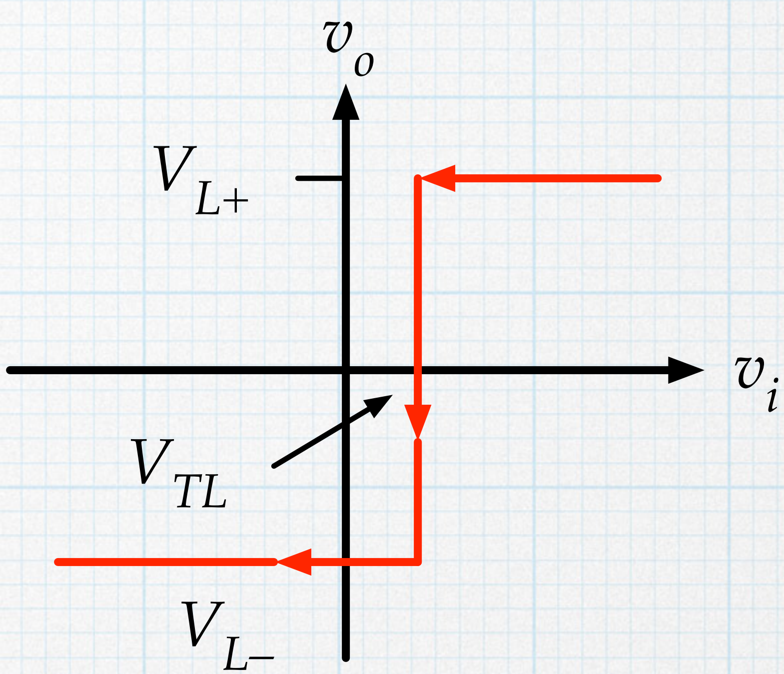
As long as $v_+ > V_{REF}$, the output will stay high.

If we start *decreasing* v_i, v_+ will decrease correspondingly. If v_i drops far enough, then v_+ will become less than V_{REF} , and the output will switch from high to low.

We can find the input voltage at which the switch occurs.

$$V_{REF} = \frac{V_{TL} + \left(\frac{R_a}{R_b}\right) V_{L+}}{1 + \frac{R_a}{R_b}} \quad \rightarrow \quad V_{TL} = V_{REF}\left(1 + \frac{R_a}{R_b}\right) - \left(\frac{R_a}{R_b}\right) V_{L+}$$

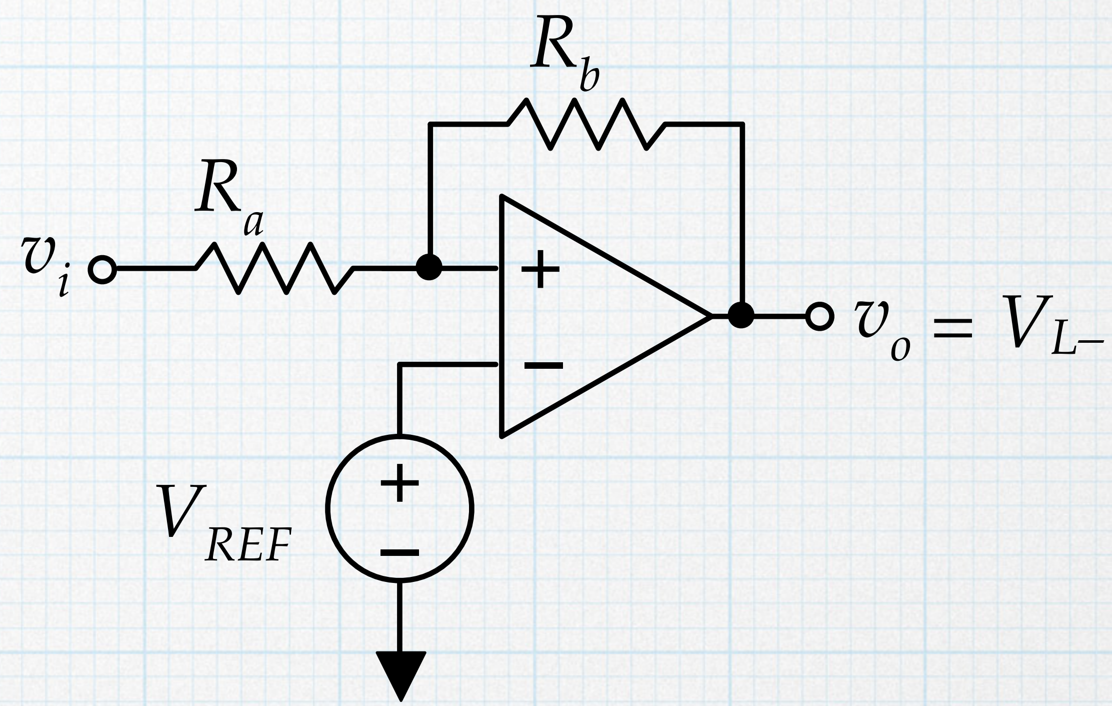




$$V_{TL} = V_{REF}\left(1 + \frac{R_a}{R_b}\right) - \left(\frac{R_a}{R_b}\right)V_{L+}$$

As v_i moves from high to low, the output will switch from high to low, as expected, and the switch occurs when the input voltage crosses V_{TL}.

Now consider the opposite case – assume that v_i is low enough to have $v_+ < V_{REF}$. Then $v_o = V_{L-}$.



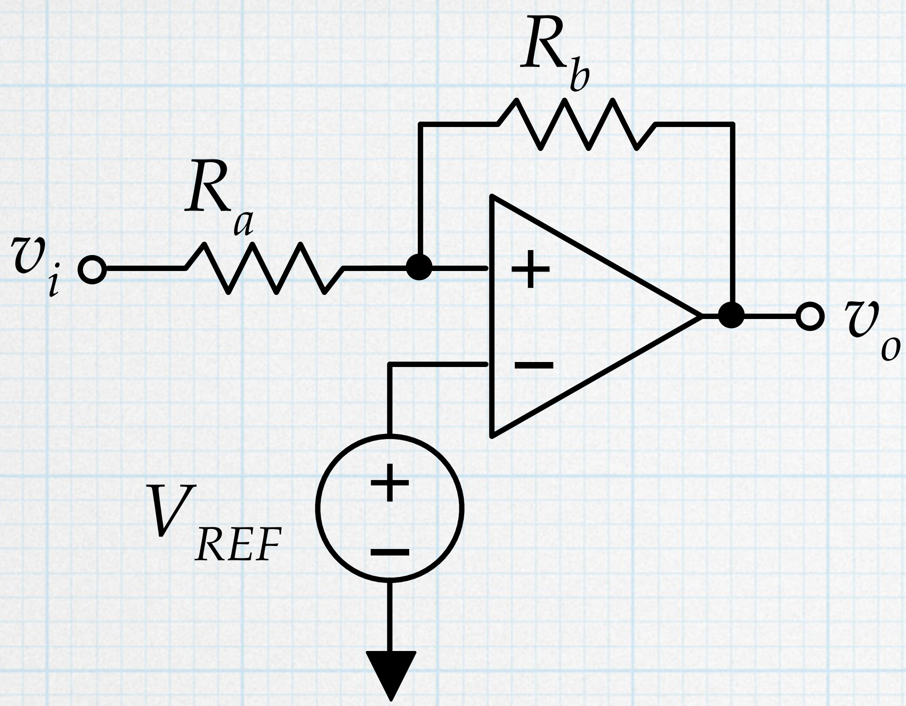
$$\frac{v_i - v_+}{R_a} = \frac{v_+ - V_{L-}}{R_b}$$

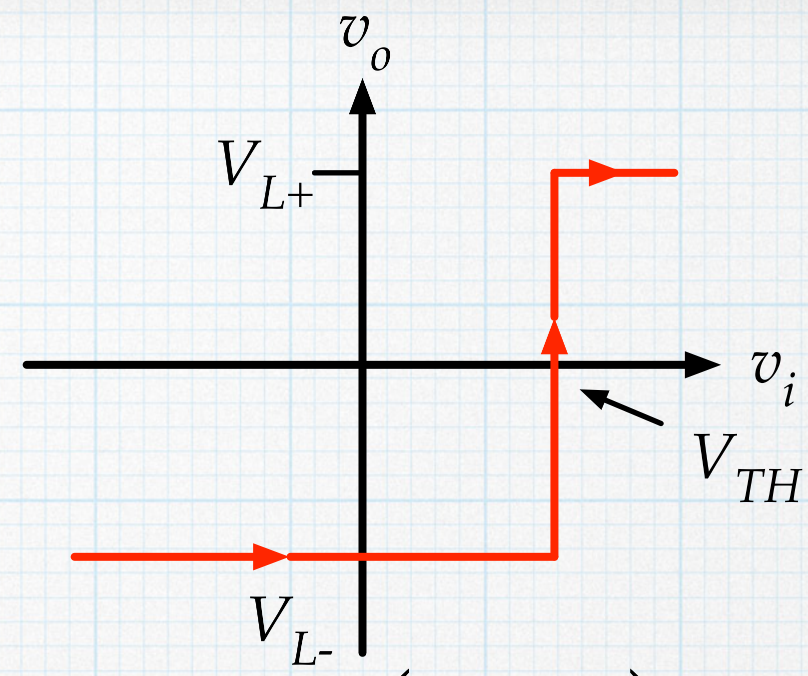
$$v_+ = \frac{v_i + \left(\frac{R_a}{R_b}\right) V_{L-}}{1 + \frac{R_a}{R_b}}$$

If we start *increasing* v_i, v_+ will increase correspondingly. If v_i increases enough, then v_+ will become greater than V_{REF} , and the output will switch from low to high.

We can find the input voltage at which the low-to-high switch occurs.

$$V_{REF} = \frac{V_{TH} + \left(\frac{R_a}{R_b}\right) V_{L-}}{1 + \frac{R_a}{R_b}} \quad \rightarrow \quad V_{TH} = V_{REF}\left(1 + \frac{R_a}{R_b}\right) - \left(\frac{R_a}{R_b}\right) V_{L-}$$

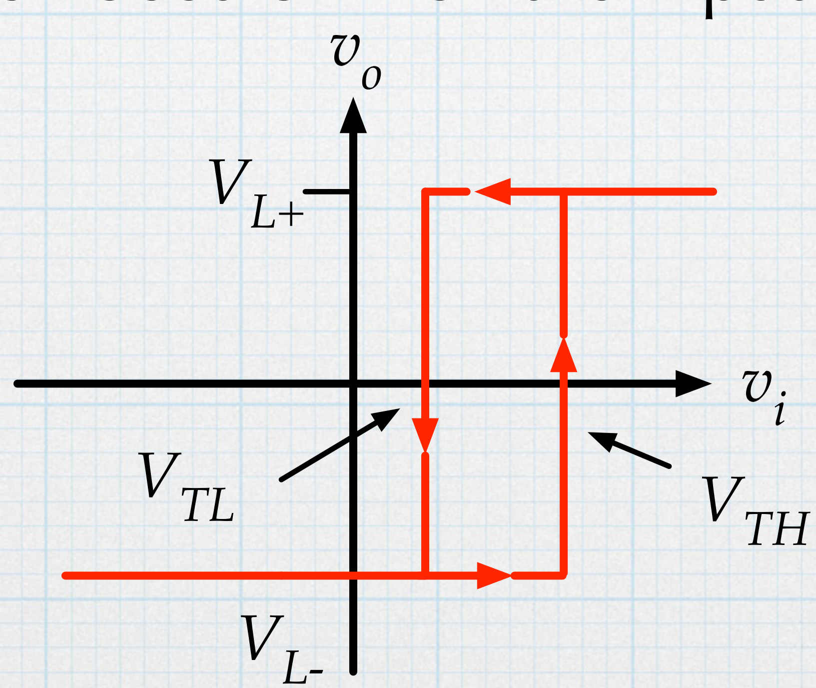




$$V_{TH} = V_{REF}\left(1 + \frac{R_a}{R_b}\right) - \left(\frac{R_a}{R_b}\right) V_{L-}$$

As v_i moves from high to low, the output will switch from high to low, as expected, and the switch occurs when the input voltage crosses V_{TH}.

Put the two together.

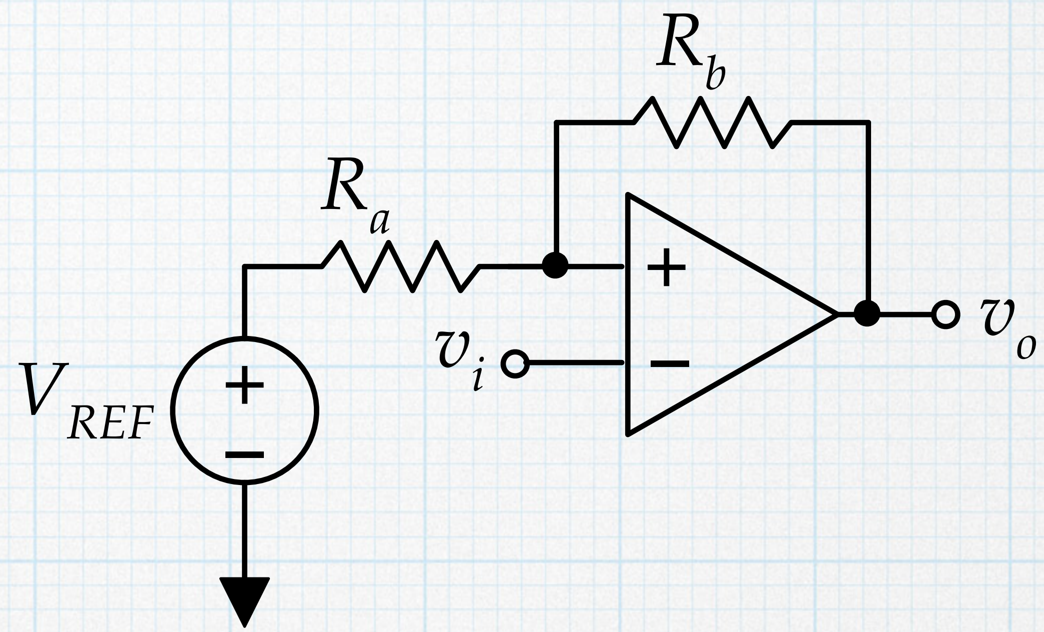


Since the up and down transitions occur at different voltages, the circuit exhibits *hysteresis*.

Hysteresis: $\Delta V_T = V_{TH} - V_{TH} = \dfrac{R_a}{R_b}\left(V_{L+} - V_{L-}\right)$

Inverting comparator with positive feedback

The same kind of thing can be done with the inverting comparator. Using positive feedback will introduce hysteresis into the transfer characteristic.



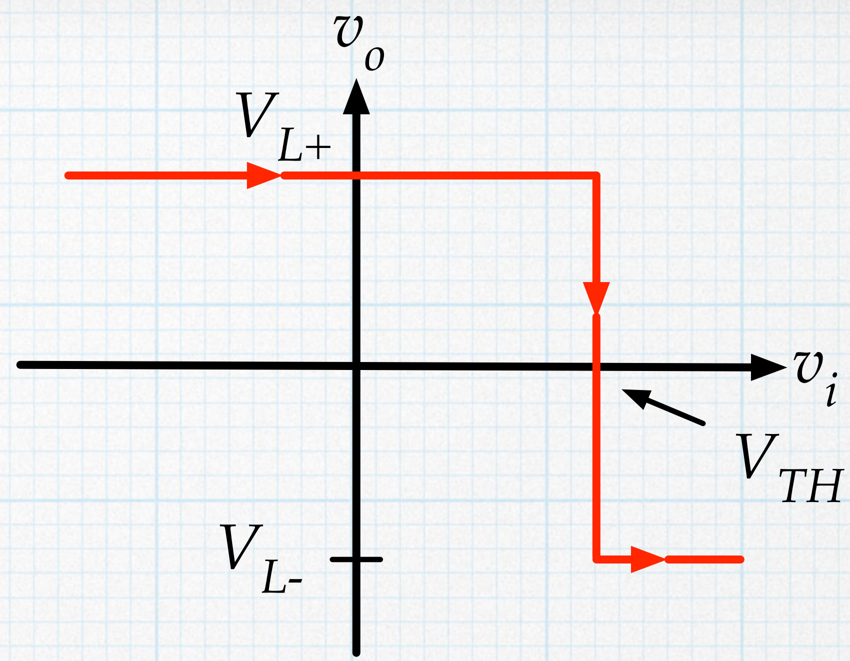
When v_i is "low" such that $v_- < v_+$, the output will be high, $v_o = V_{L+}$. As v_i increases, at some point it becomes bigger than v_+ and the output will switch low. So we need to know the value of v_+ when the output is high. Writing the node equation at the non-inverting terminal,

$$\frac{V_{REF} - v_+}{R_a} = \frac{v_+ - V_{L+}}{R_b}$$

Solve for v_+ : $$v_+ = \frac{V_{REF} + \left(\frac{R_a}{R_b}\right) V_{L+}}{1 + \frac{R_a}{R_b}}$$

When v_i crosses this value, the output will switch from high to low.

$$V_{TH} = \frac{V_{REF} + \left(\frac{R_a}{R_b}\right) V_{L+}}{1 + \frac{R_a}{R_b}}$$



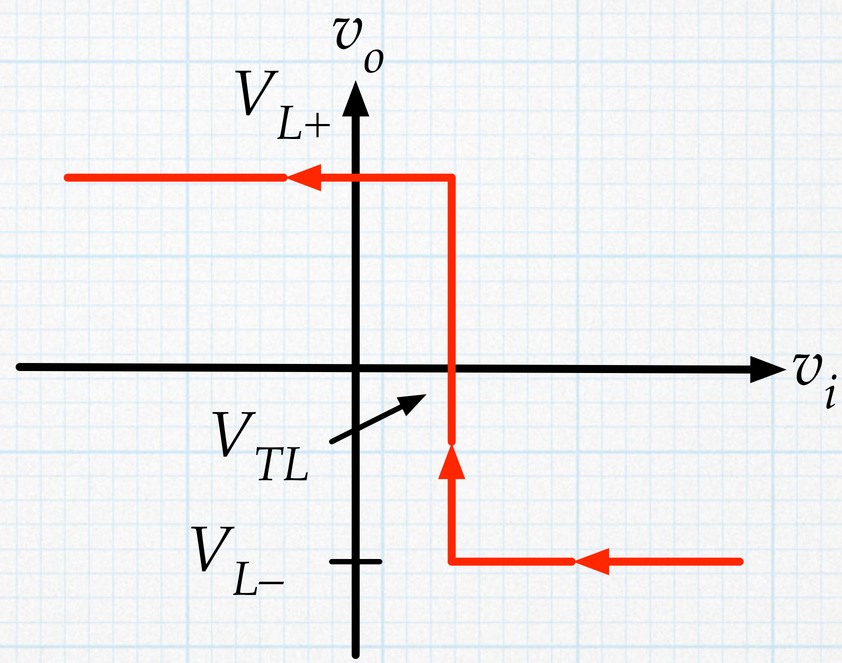
Now consider the opposite case. When v_i is "high" (so that $v_- > v_+$), the output will low, $v_o = V_{L-}$. As v_i decreases, at some point it becomes smaller than v_+ and the output will go high. So we need to know the value of v_+ when the output is low. Again, writing the node equation,

$$\frac{V_{REF} - v_+}{R_a} = \frac{v_+ - V_{L-}}{R_b}$$

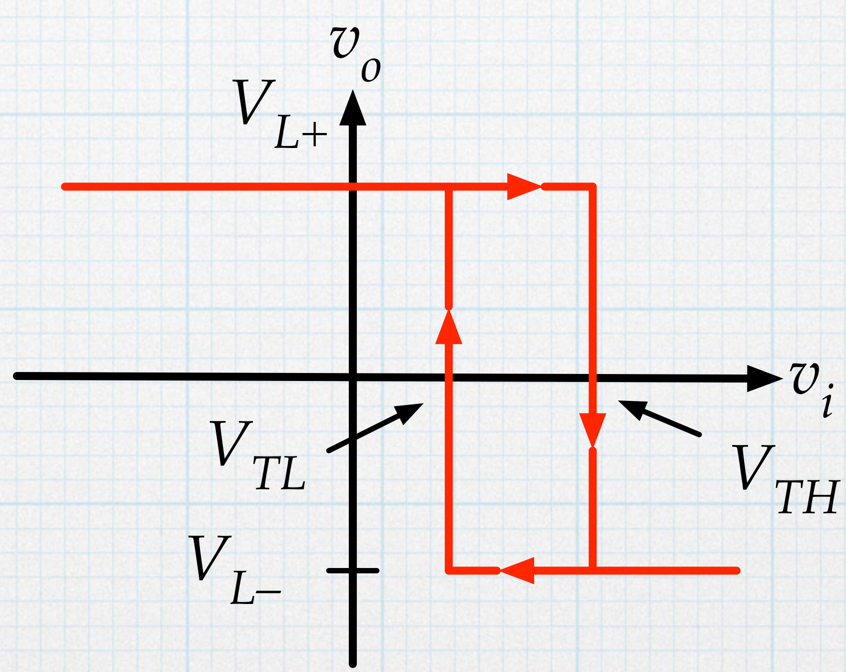
Solving for v_+ : $\quad v_+ = \frac{V_{REF} + \left(\frac{R_a}{R_b}\right) V_{L-}}{1 + \frac{R_a}{R_b}}$

When v_i crosses this value, the output will switch from low to high.

$$V_{TL} = \frac{V_{REF} + \left(\frac{R_a}{R_b}\right) V_{L-}}{1 + \frac{R_a}{R_b}}$$



Put the two together.

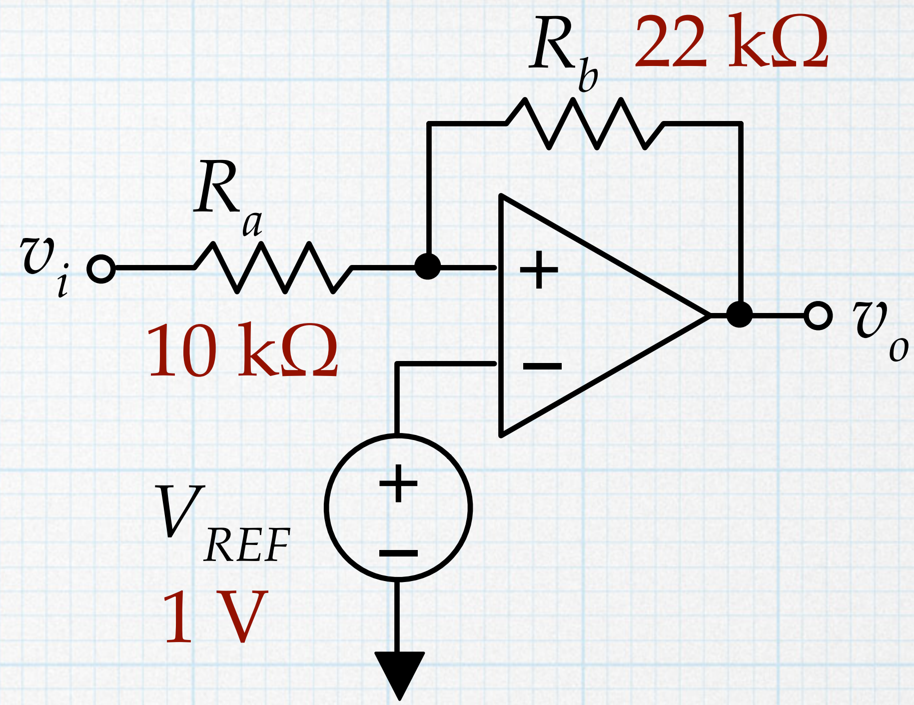


Hysteresis: $$\Delta V_T = V_{TH} - V_{TL} = \frac{\left(\frac{R_a}{R_b}\right)\left(V_{L+} - V_{L-}\right)}{1 + \frac{R_a}{R_b}} = \frac{V_{L+} - V_{L-}}{1 + \frac{R_b}{R_a}}$$

Example

Find V_{TL} and V_{TH} for the non-inverting comparator shown. Also calculate the hysteresis width for the comparator.

The op-amp has high and low output limits of V_{L+} = +7.5 V and V_{L-} = –7.5 V.



$$V_{TL} = V_{REF}\left(1 + \frac{R_a}{R_b}\right) - \left(\frac{R_a}{R_b}\right) V_{L+}$$

$$= (1\ \text{V})\left(1 + \frac{10\ \text{k}\Omega}{22\ \text{k}\Omega}\right) - \left(\frac{10\ \text{k}\Omega}{22\ \text{k}\Omega}\right)(7.5\ \text{V}) = -1.95\ \text{V}$$

$$V_{TH} = V_{REF}\left(1 + \frac{R_a}{R_b}\right) - \left(\frac{R_a}{R_b}\right) V_{L-}$$

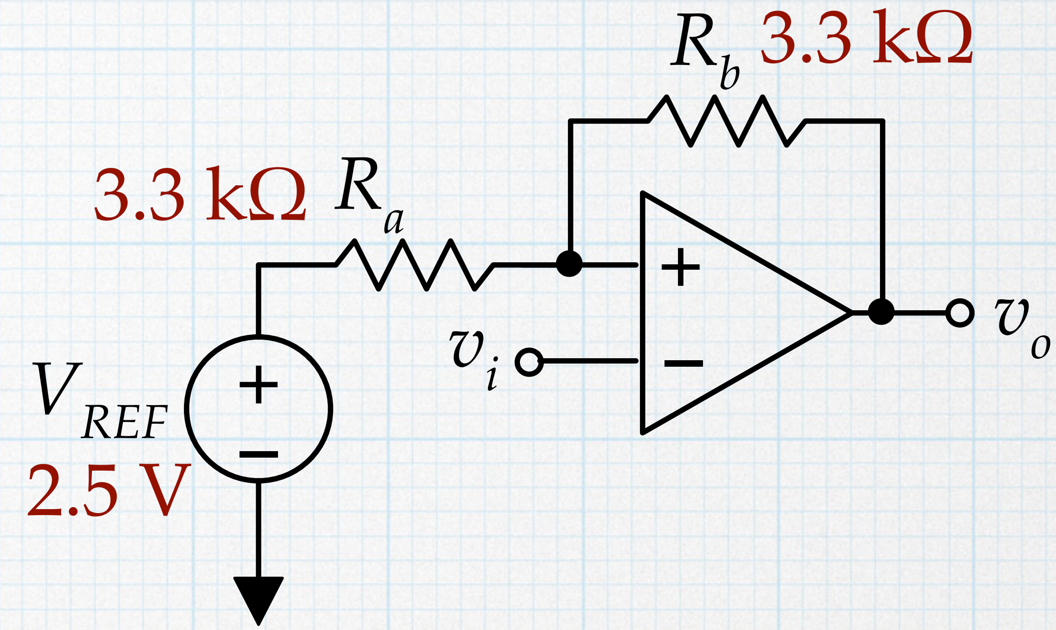
$$= (1\ \text{V})\left(1 + \frac{10\ \text{k}\Omega}{22\ \text{k}\Omega}\right) - \left(\frac{10\ \text{k}\Omega}{22\ \text{k}\Omega}\right)(-7.5\ \text{V}) = 4.86\ \text{V}$$

$$\Delta V = \frac{R_a}{R_b}\left(V_{L+} - V_{L-}\right) = 6.81\ \text{V}$$

Example

Find V_{TL} and V_{TH} for the inverting comparator shown. Also calculate the hysteresis width for the comparator.

The op-amp works with a single supply and has high and low output limits of V_{L+} = +6 V and V_{L-} = 0 V.

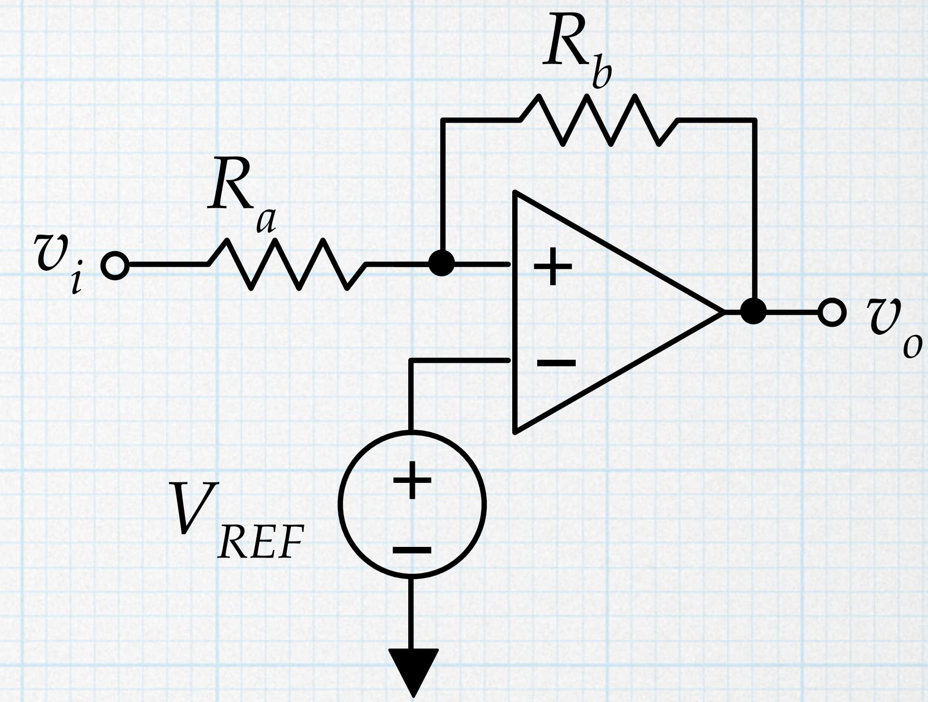


$$V_{TL} = \frac{V_{REF} + \frac{R_a}{R_b} V_{L-}}{1 + \frac{R_a}{R_b}} = \frac{2.5\text{ V} + \left(\frac{3.3\text{ k}\Omega}{3.3\text{ k}\Omega}\right)(0\text{ V})}{1 + \left(\frac{3.3\text{ k}\Omega}{3.3\text{ k}\Omega}\right)} = 1.25\text{ V}$$

$$V_{TH} = \frac{V_{REF} + \frac{R_a}{R_b} V_{L+}}{1 + \frac{R_a}{R_b}} = \frac{2.5\text{ V} + \left(\frac{3.3\text{ k}\Omega}{3.3\text{ k}\Omega}\right)(6\text{ V})}{1 + \left(\frac{3.3\text{ k}\Omega}{3.3\text{ k}\Omega}\right)} = 4.25\text{ V}$$

$$\Delta V = \frac{V_{L+} - V_{L-}}{1 + \frac{R_a}{R_b}} = \frac{6\text{ V} - 0}{1 + \left(\frac{3.3\text{ k}\Omega}{3.3\text{ k}\Omega}\right)} = 3\text{ V}$$

Example



Design the non-inverting comparator shown at right so that it has $V_{TL} = 2.0$ V and $V_{TH} = 4.0$ V

The op-amp has high and low output limits of $V_{L+} = +6$ V and $V_{L-} = 0$.

$$V_{TH} = V_{REF}\left(1 + \frac{R_a}{R_b}\right) - \left(\frac{R_a}{R_b}\right)V_{L-}$$

$$V_{TL} = V_{REF}\left(1 + \frac{R_a}{R_b}\right) - \left(\frac{R_a}{R_b}\right)V_{L+}$$

Subtracting the bottom equation from the top gives:

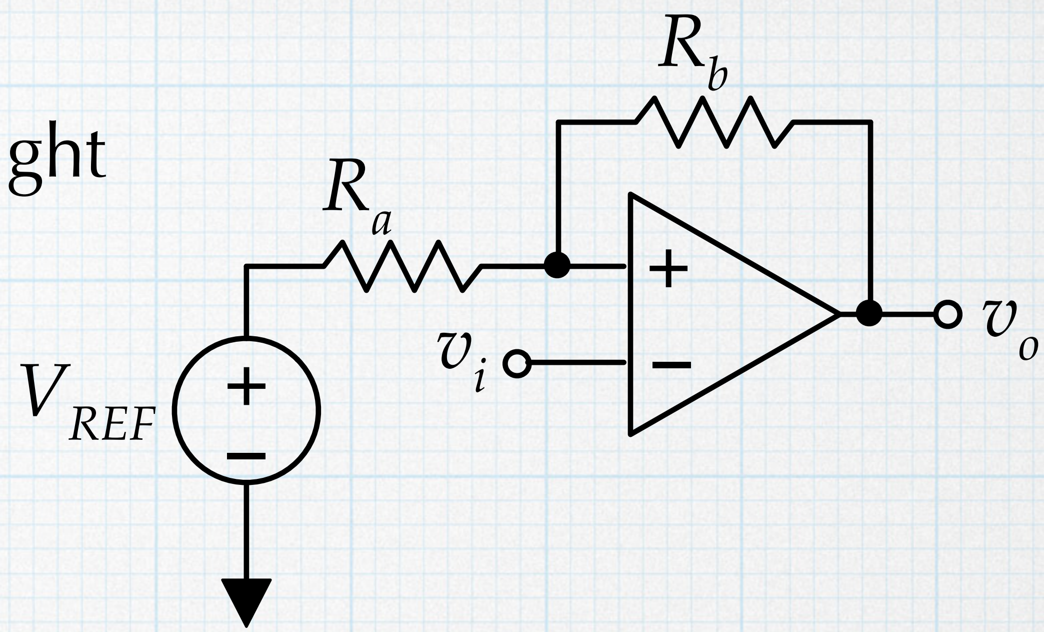
$$V_{TH} - V_{TL} = \frac{R_a}{R_b}\left(V_{L+} - V_{L-}\right) \longrightarrow \frac{R_a}{R_b} = \frac{V_{TH} - V_{TL}}{V_{L+} - V_{L-}} = \frac{4\text{ V} - 2\text{ V}}{6\text{ V}} = 0.333$$

Choose a pair resistors with this ratio. Then, using the V_{TL} equation:

$$V_{REF} = \frac{V_{TL} + \frac{R_a}{R_b}V_{L+}}{1 + \frac{R_a}{R_b}} = \frac{2\text{ V} + (0.333)\,(6\text{ V})}{1 + (0.333)} = 3\text{ V}$$

(Using V_{TH} equation would give the same answer.)

Example



Design the inverting comparator shown at right so that it has V_{TL} = –1.0 V and V_{TH} = +3.0 V

The op-amp has high and low output limits of V_{L+} = +5 V and V_{L-} = –5 V.

$$V_{TH} = \frac{V_{REF} + \frac{R_a}{R_b} V_{L+}}{1 + \frac{R_a}{R_b}} \qquad V_{TL} = \frac{V_{REF} + \frac{R_a}{R_b} V_{L-}}{1 + \frac{R_a}{R_b}}$$

Subtracting the right equation from the left one gives:

$$V_{TH} - V_{TL} = \frac{\left(\frac{R_a}{R_b}\right)(V_{L+} - V_{L-})}{1 + \frac{R_a}{R_b}}$$

After a bit of algebra (note the inversion of the ratio):

$$\frac{R_b}{R_a} = \frac{V_{L+} - V_{L-}}{V_{TH} - V_{TL}} - 1 = \frac{5\text{ V} - (-5\text{ V})}{3\text{ V} - (-1\text{ V})} - 1 = 1.5$$

or $$\frac{R_a}{R_b} = 0.667$$

Finally, using the V_{TH} equation:

$$V_{REF} = V_{TH}\left(1 + \frac{R_a}{R_b}\right) - \left(\frac{R_a}{R_b}\right) V_{L+}$$

$$= (3\text{ V})(1 + 0.667) - (0.667)(5\text{ V}) = 1.67\text{ V}$$